



Geophysical Investigation of Asteroids by Dawn Spacecraft

Caltech Planetary Seminar

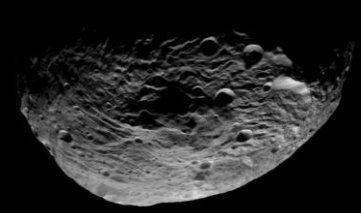
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³University of California Los Angeles

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Goal of the talk:



- Explain how the internal structures of **Vesta** and **Ceres** evolved by looking at the present-day topography and gravity measured by **Dawn**



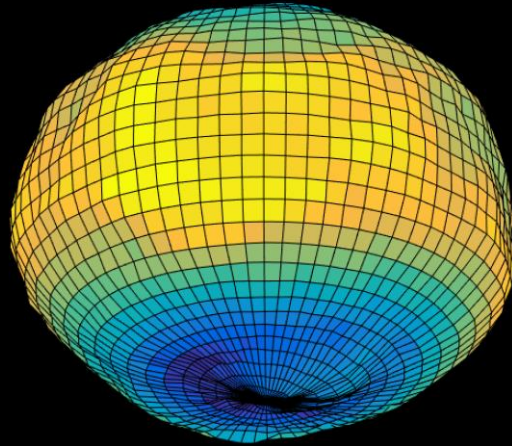
How do we use shape data to study interiors?



- **Hydrostatic equilibrium**
 - **Isostatic compensation**
 - **Viscous relaxation**
 - **Shape model is required for computing gravity anomalies**
 - **Topographic roughness**
 - **Local geomorphology**
- } **beyond this talk**

Shape models

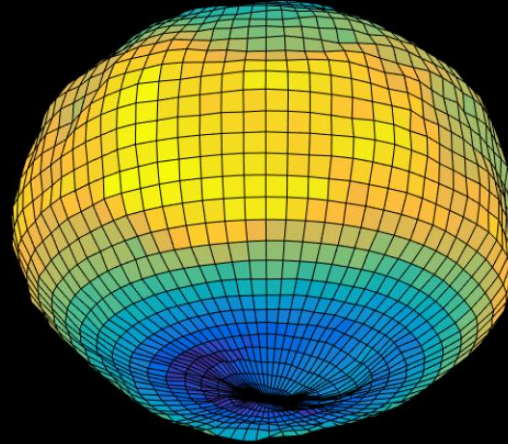
➤ Geographic grid



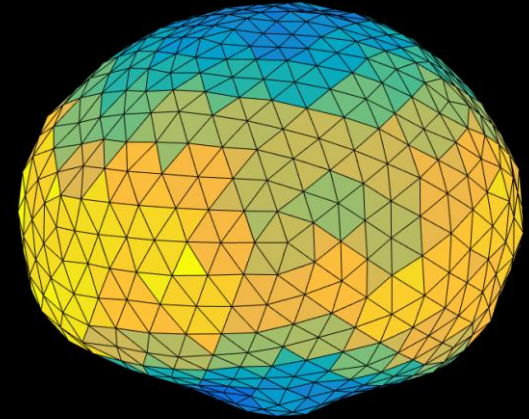
Shape models



➤ Geographic grid



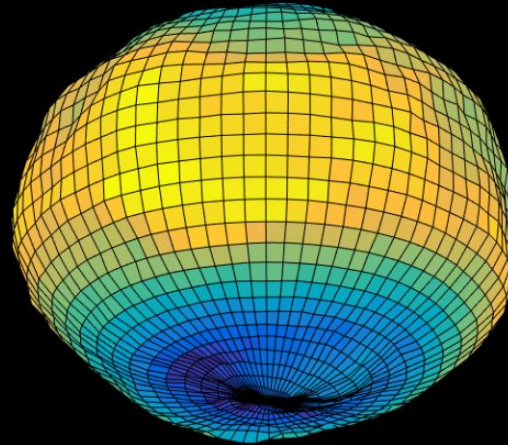
➤ Polyhedral model



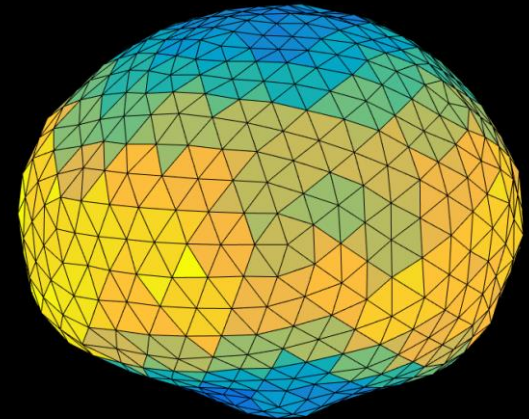
Shape models



➤ Geographic grid

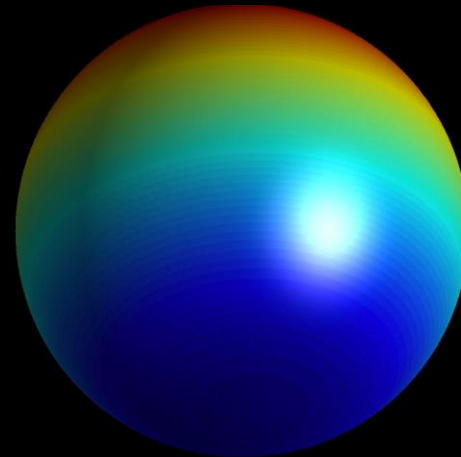


➤ Polyhedral model



➤ Spherical harmonic expansion

- set of orthogonal functions on a sphere



Gravity models

- Spherical harmonics

$$U(r, \varphi, \lambda) = \frac{GM}{r} \left[1 + \sum_{n=2}^{\infty} \left(\frac{R_0}{r} \right)^n \left(C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right) P_n(\sin \varphi) \right]$$

U – gravitational potential

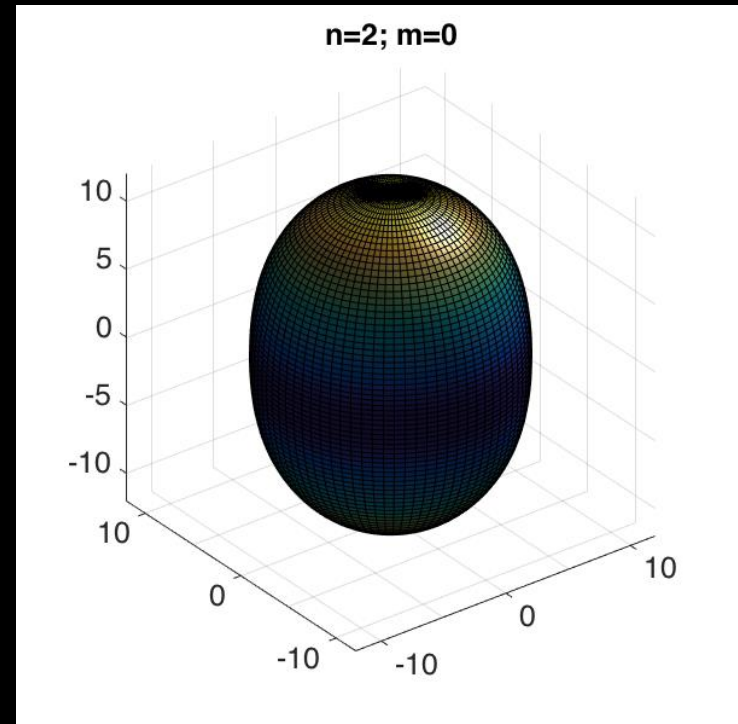
φ – latitude

λ – longitude

r – radial distance

n – degree

m – order



Gravity models

- Spherical harmonics

$$U(r, \varphi, \lambda) = \frac{GM}{r} \left[1 + \sum_{n=2}^{\infty} \left(\frac{R_0}{r} \right)^n \left(C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right) P_n(\sin \varphi) \right]$$

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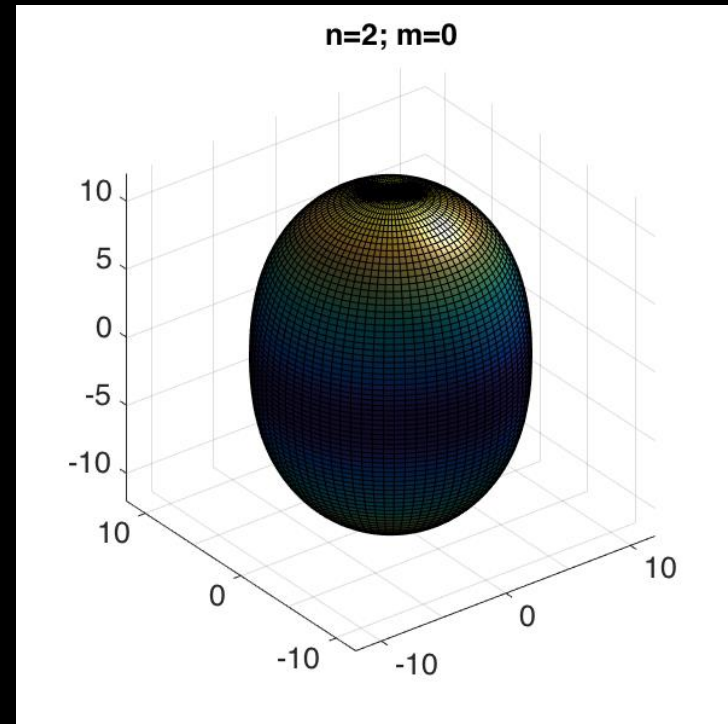
λ – longitude

r – radial distance

n – degree

m – order

- Ellipsoidal harmonics
- Mascons



Gravity and topography in spherical harmonics

- Shape radius vector

$$r(f, l) = R_0 \sum_{n=1}^{\infty} \sum_{m=0}^n (A_{nm} \cos(m l) + B_{nm} \sin(m l)) P_{nm}(\sin f)$$

- Gravitational potential

$$U(r, f, l) = \frac{GM}{R} + \sum_{n=2}^{\infty} \sum_{m=0}^n \frac{R_0^n}{r^{n+1}} (C_{nm} \cos(m l) + S_{nm} \sin(m l)) P_{nm}(\sin f)$$

- Power Spectral Density

$$S_n^{gg} = \sum_{m=0}^n \frac{C_{nm}^2 + S_{nm}^2}{2n+1}$$

gravity

$$S_n^{tt} = \sum_{m=0}^n \frac{A_{nm}^2 + B_{nm}^2}{2n+1}$$

topography

$$S_n^{gt} = \sum_{m=0}^n \frac{A_{nm} C_{nm} + B_{nm} S_{nm}}{2n+1}$$

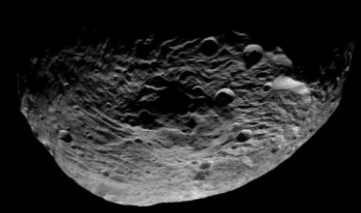
gravity-topography
cross power



Hydrostatic equilibrium



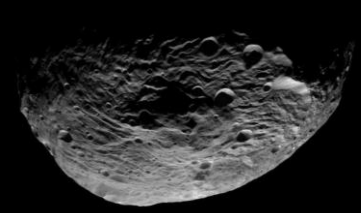
- **In** hydrostatic equilibrium
 - Surfaces of constant density, pressure and potential coincide
 - No shear stresses



Hydrostatic equilibrium



➤ **In** hydrostatic equilibrium



Hydrostatic equilibrium

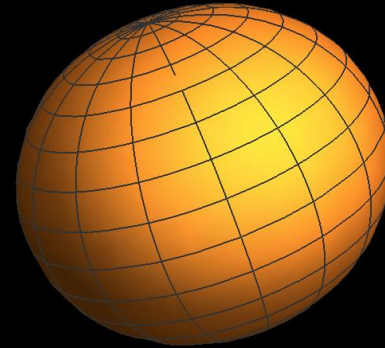
➤ In hydrostatic equilibrium

$$\rho = \rho(r), \omega$$

Hydrostatic equilibrium

➤ In hydrostatic equilibrium

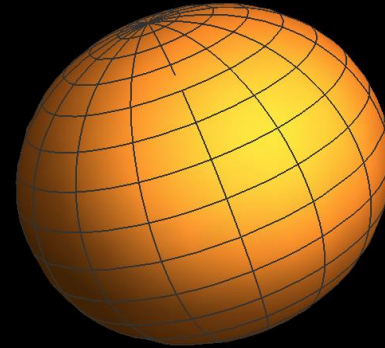
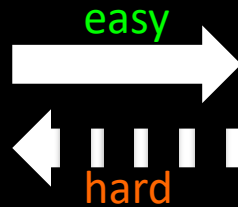
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Hydrostatic equilibrium

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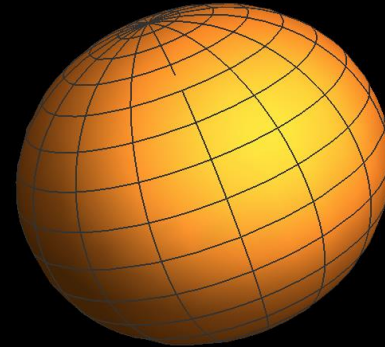
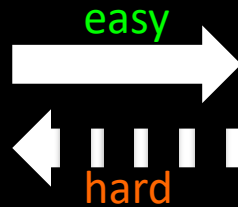




Hydrostatic equilibrium

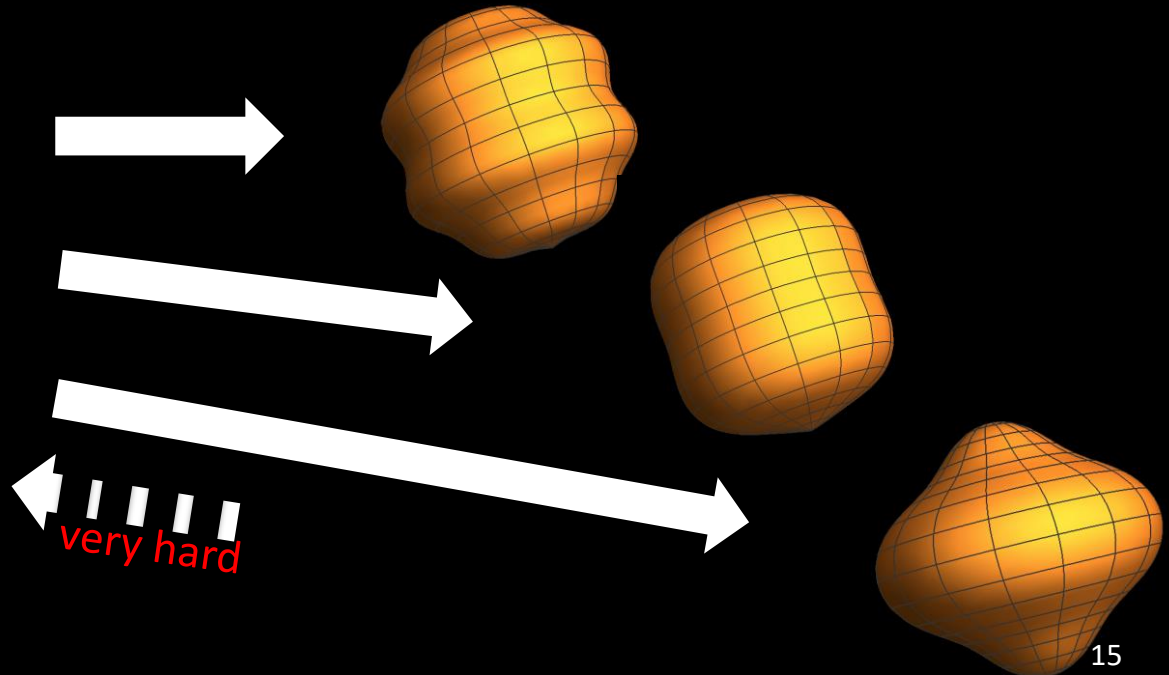
➤ **In** hydrostatic equilibrium

$$\rho = \rho(r), \omega$$

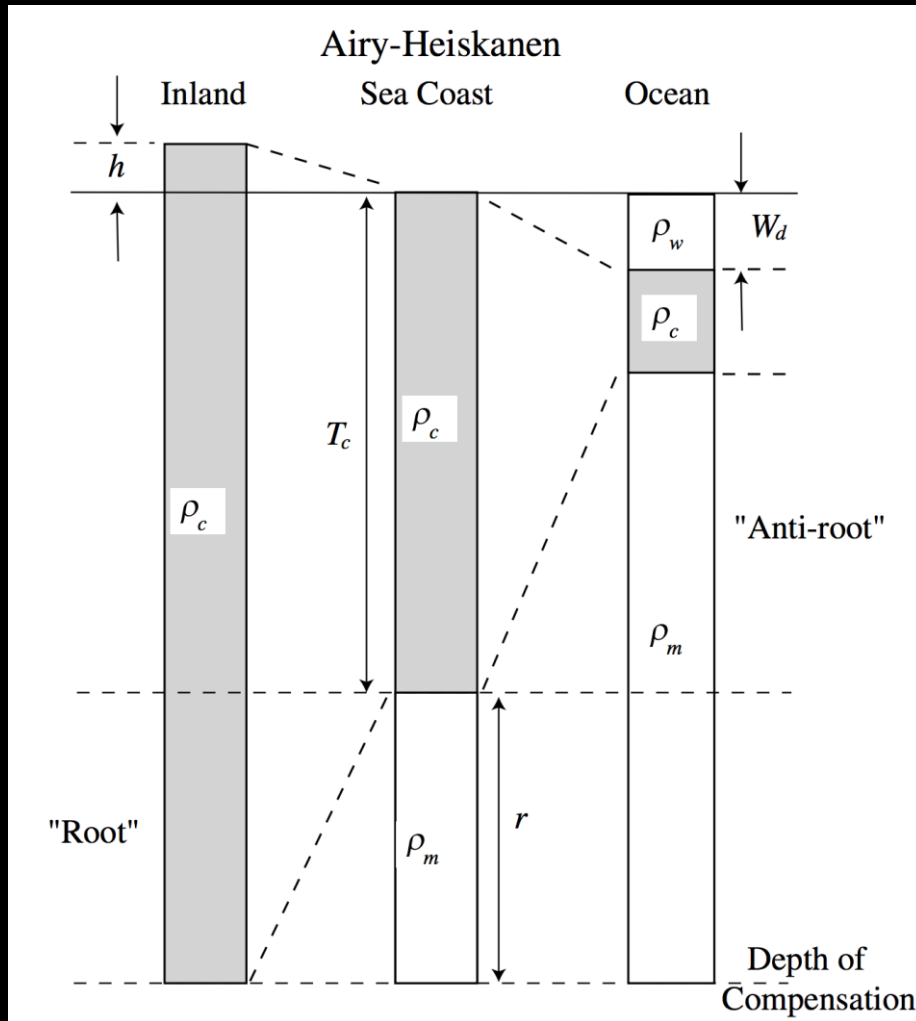


➤ **Not in** hydrostatic equilibrium

$$\rho = \rho(r), \omega$$



Isostasy



Watts, 2001

Isostatic equilibrium:

- Equal weight of crustal columns at the depth of compensation
- Deviatoric stresses within the isostatically compensated layer are minimized

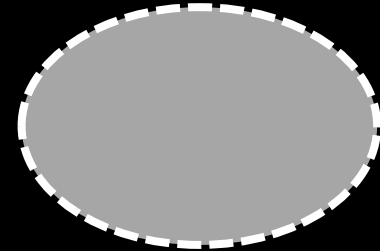
Gravity anomalies

- Free-air anomaly

$$\sigma_{\text{FA}} = \sigma_{\text{obs}} - \sigma_{\text{model}}$$

$$\sigma_{\text{model}} =$$

gravity of
hydrostatic figure





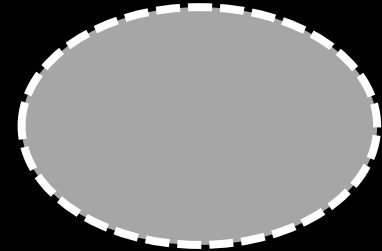
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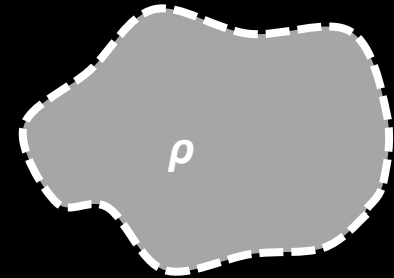


- Bouguer anomaly

$$\sigma_{\text{BA}} = \sigma_{\text{obs}} - \sigma_{\text{model}}$$

$$\sigma_{\text{model}} =$$

gravity of shape
assuming ρ

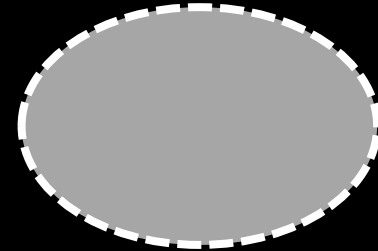


Gravity anomalies

- Free-air anomaly

$$\sigma_{FA} = \sigma_{obs} - \sigma_{model}$$

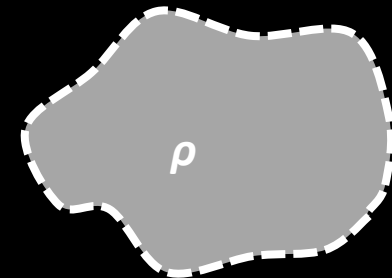
$\sigma_{model} =$ gravity of hydrostatic figure



- Bouguer anomaly

$$\sigma_{BA} = \sigma_{obs} - \sigma_{model}$$

$\sigma_{model} =$ gravity of shape assuming ρ

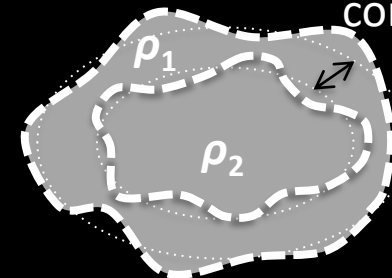


- Isostatic anomaly

$$\sigma_{IA} = \sigma_{obs} - \sigma_{model}$$

h – depth of compensation

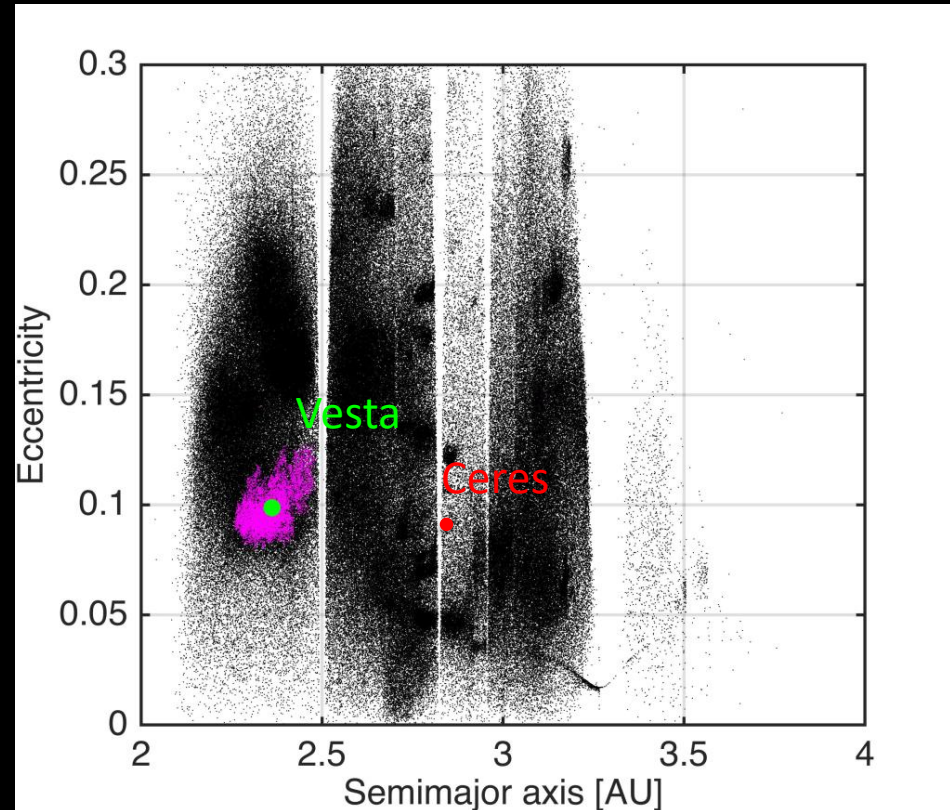
$\sigma_{model} =$ gravity assuming isostasy for ρ_1, ρ_2, h



Why Ceres?

- Largest body in the asteroid belt
- Low density implies high volatile content
- Conditions for subsurface ocean
- Much easier to reach than other ocean worlds

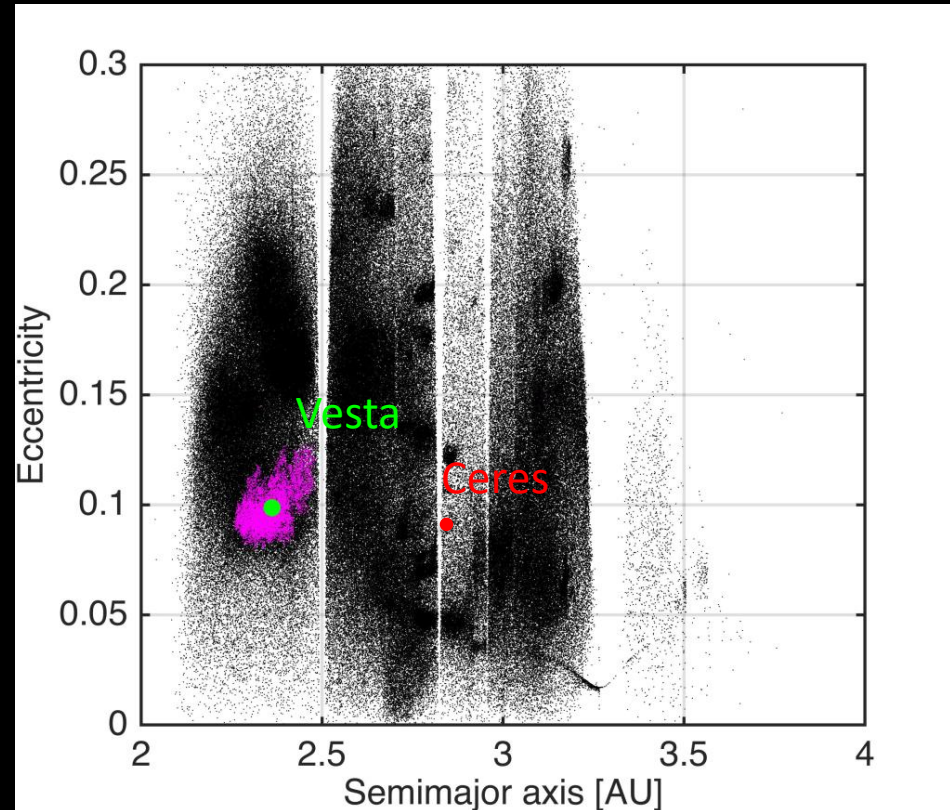
Ceres location in the asteroid belt



Why Ceres?

- **Largest body in the asteroid belt**
- **Low density implies high volatile content**
- **Conditions for subsurface ocean**
- **Much easier to reach than other ocean worlds**
- **Major unexplored object in the asteroid belt**

Ceres location in the asteroid belt





What did we know before Dawn



- **Castillo-Rogez and McCord 2010**

Ceres accreted as a mixture of ice and rock just a few My after the condensation of Calcium Aluminum-rich Inclusions (CAIs), and later differentiated into a water mantle and a mostly anhydrous silicate core.



What did we know before Dawn

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- **Zolotov 2009**

Ceres formed relatively late from planetesimals consisting of hydrated silicates.



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Ceres formed relatively late from planetesimals consisting of hydrated silicates.

- **Bland 2013**

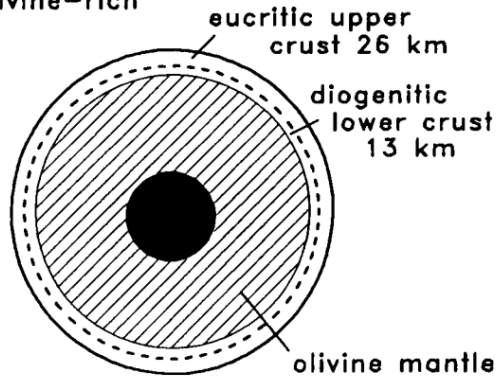
If Ceres *does* contain a water ice layer, its warm diurnally-averaged surface temperature ensures extensive viscous relaxation of even small impact craters especially near equator

What did we know before Dawn?

Vesta

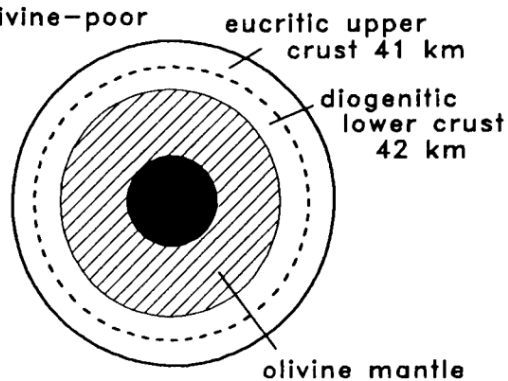
Vesta

olivine-rich



Vesta

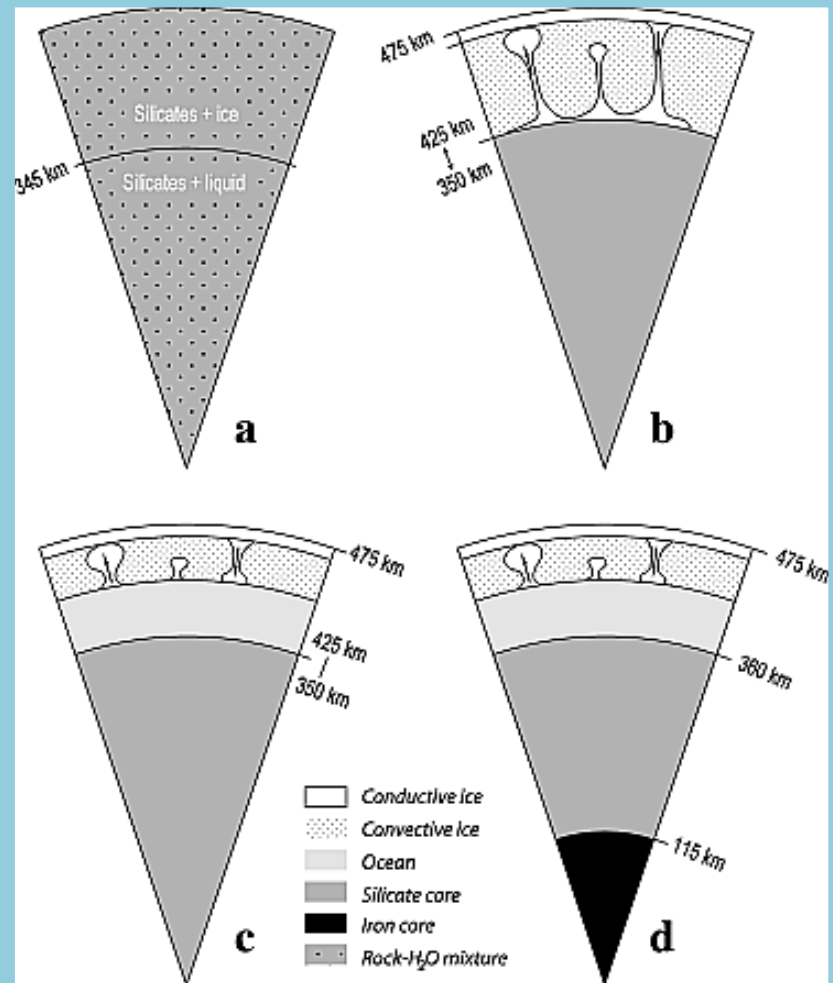
olivine-poor



core mass = 5%
core radius = 75 km
asteroid radius = 265 km

Ruzicka et al., 1997

Ceres



McCord and Sotin, 2005

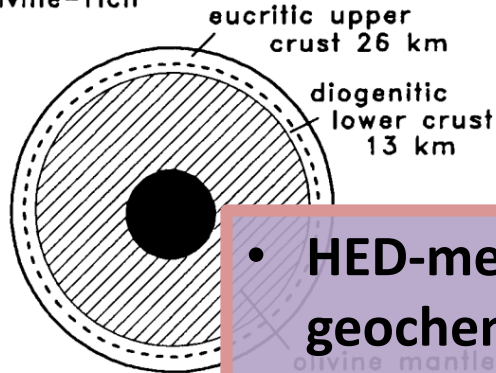
What did we know before Dawn?

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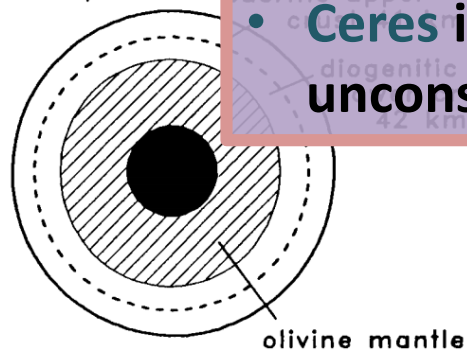
Vesta

olivine-rich



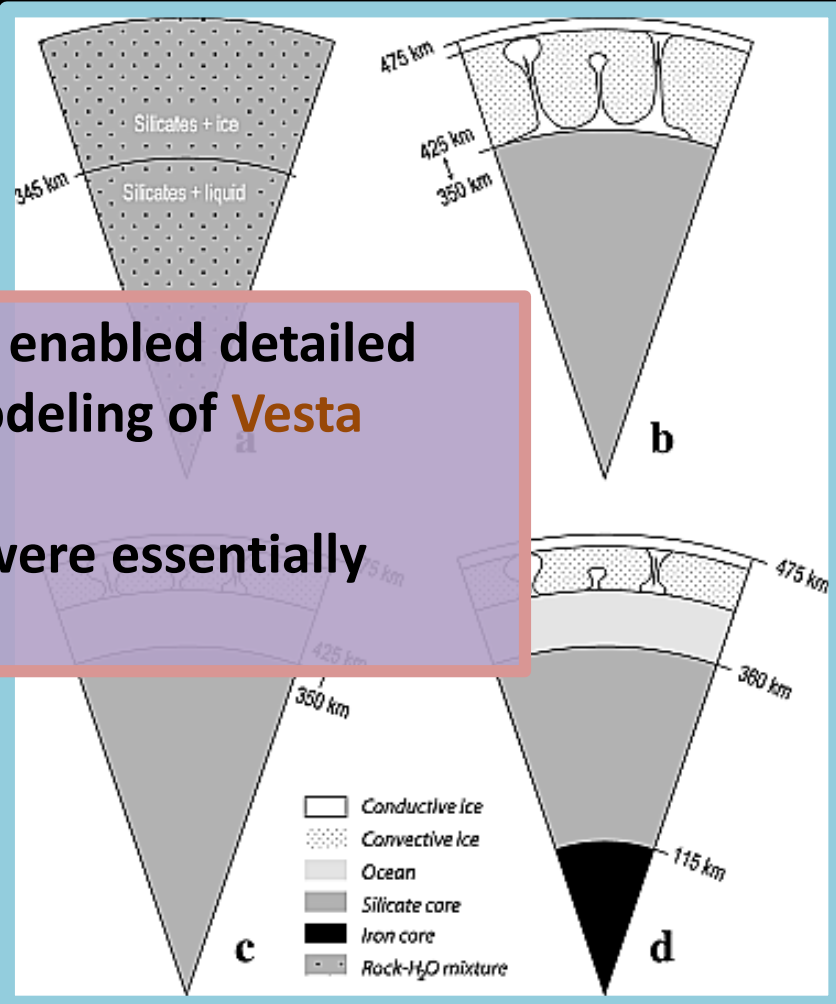
Vesta

olivine-poor



core mass = 5%
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- HED-meteorites enabled detailed geochemical modeling of **Vesta**
- **Ceres** interiors were essentially unconstrained



Ruzicka et al., 1997

McCord and Sotin, 2005

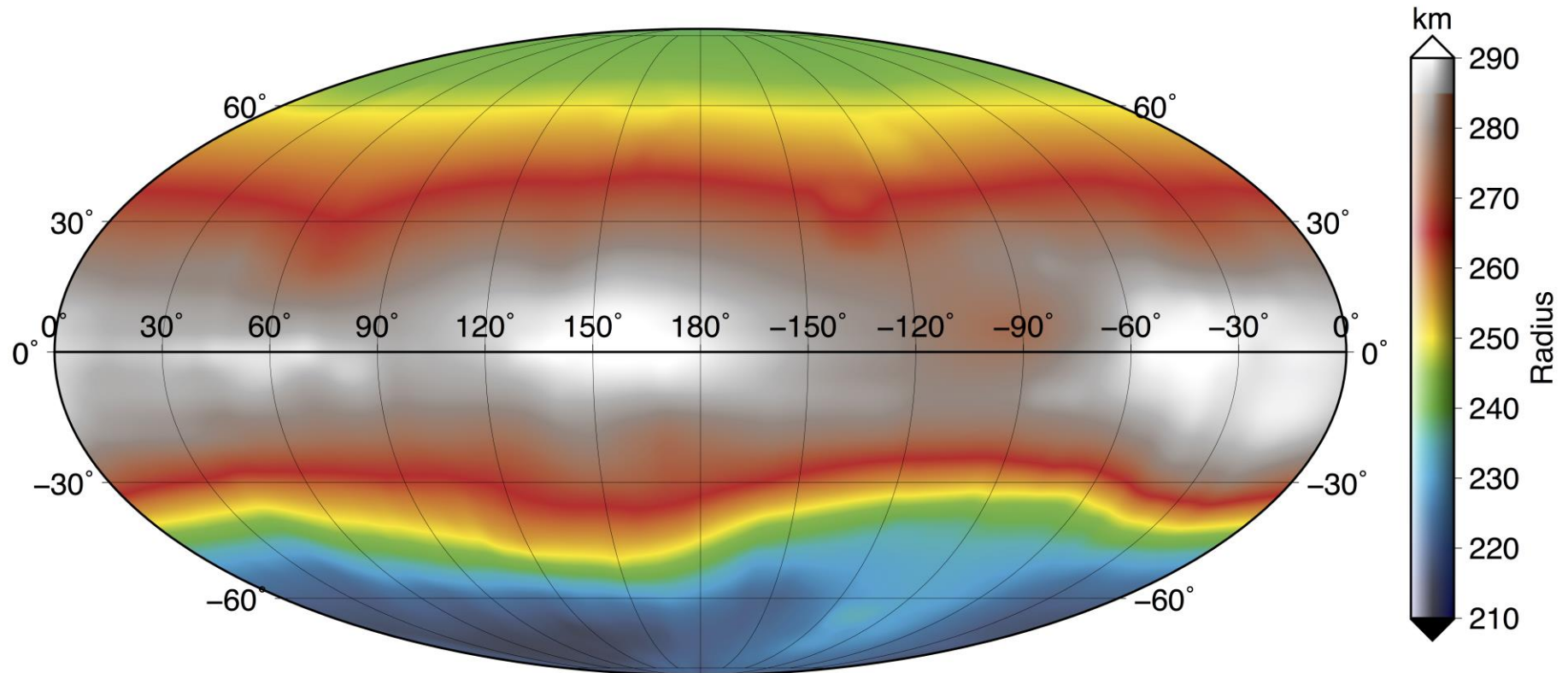


Dawn *geophysical* data

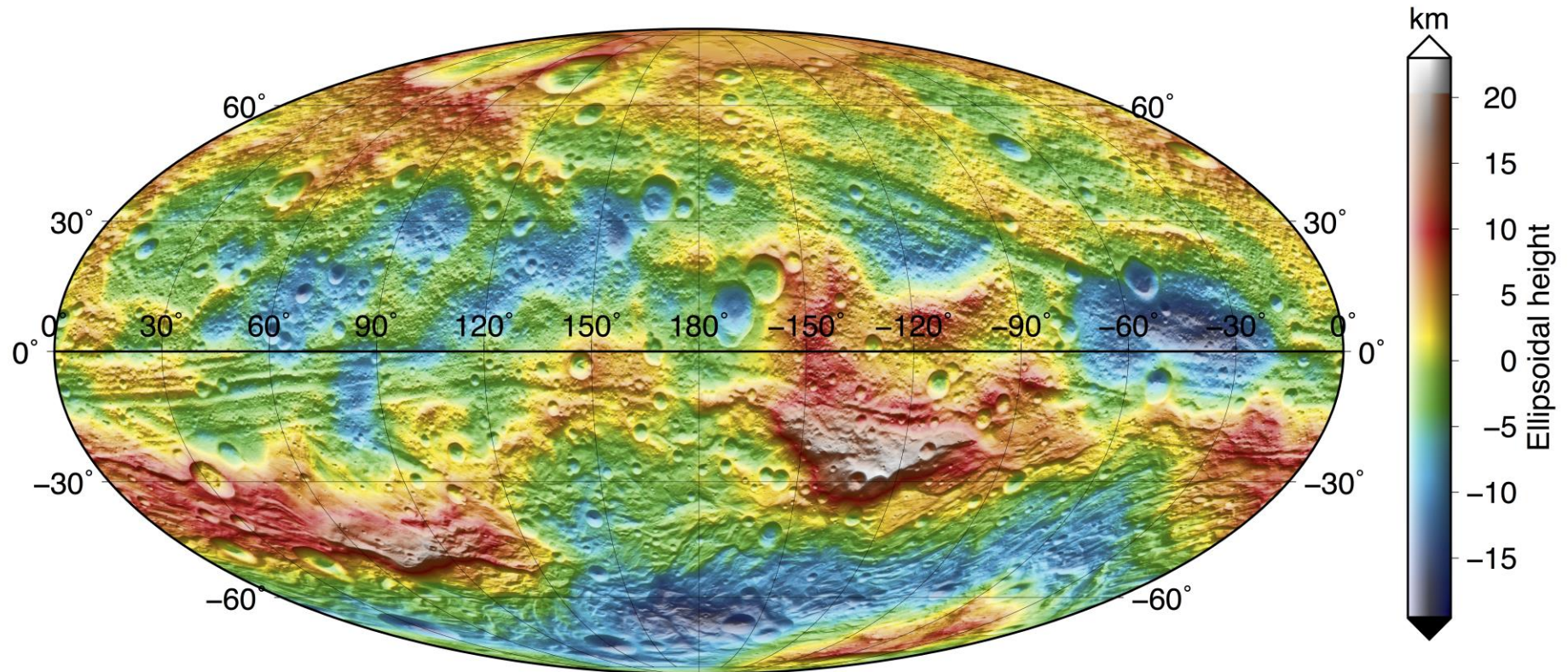


- Shape model
 - Stereophotogrammetry (SPG) from DLR
 - Stereophotoclinometry (SPC) from JPL
 - Mutually consistent with the accuracy much better than the spatial resolution of gravity field
- Gravity field
 - Accurate up to $n = 18$ ($\lambda=93$ km) for **Vesta** (Konopliv et al., 2014)
 - Accurate up to $n = 17$ ($\lambda=174$ km) for **Ceres** (Konopliv et al., 2017)
- Assumptions we have to make:
 - Multilayer model with uniform density layers
 - Range of core densities for **Vesta**
 - Range of crustal densities from HEDs for **Vesta**
 - Can't really assume anything for **Ceres**

Vesta (Thomas et al, 1997)



Vesta SPC

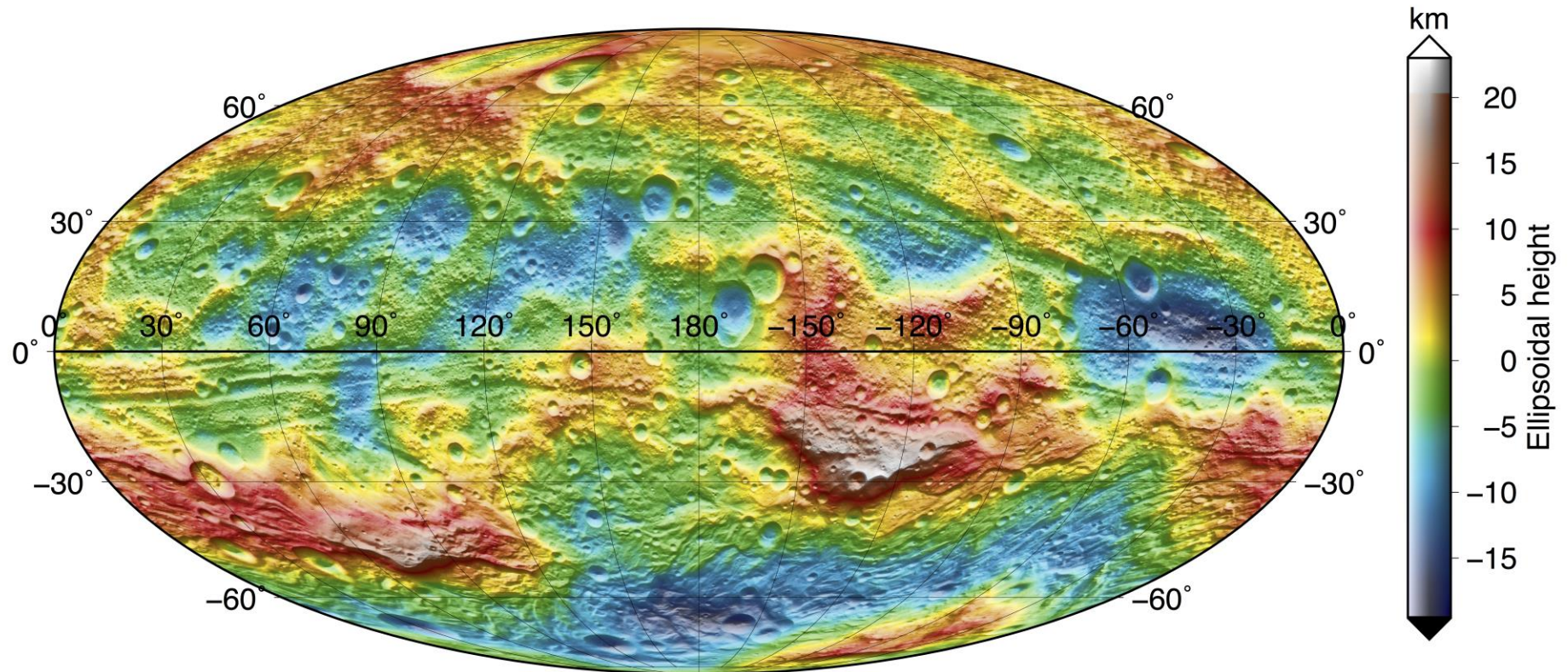


Reference ellipsoid:

$a = 280.9$ km

$c = 226.2$ km

Vesta SPG

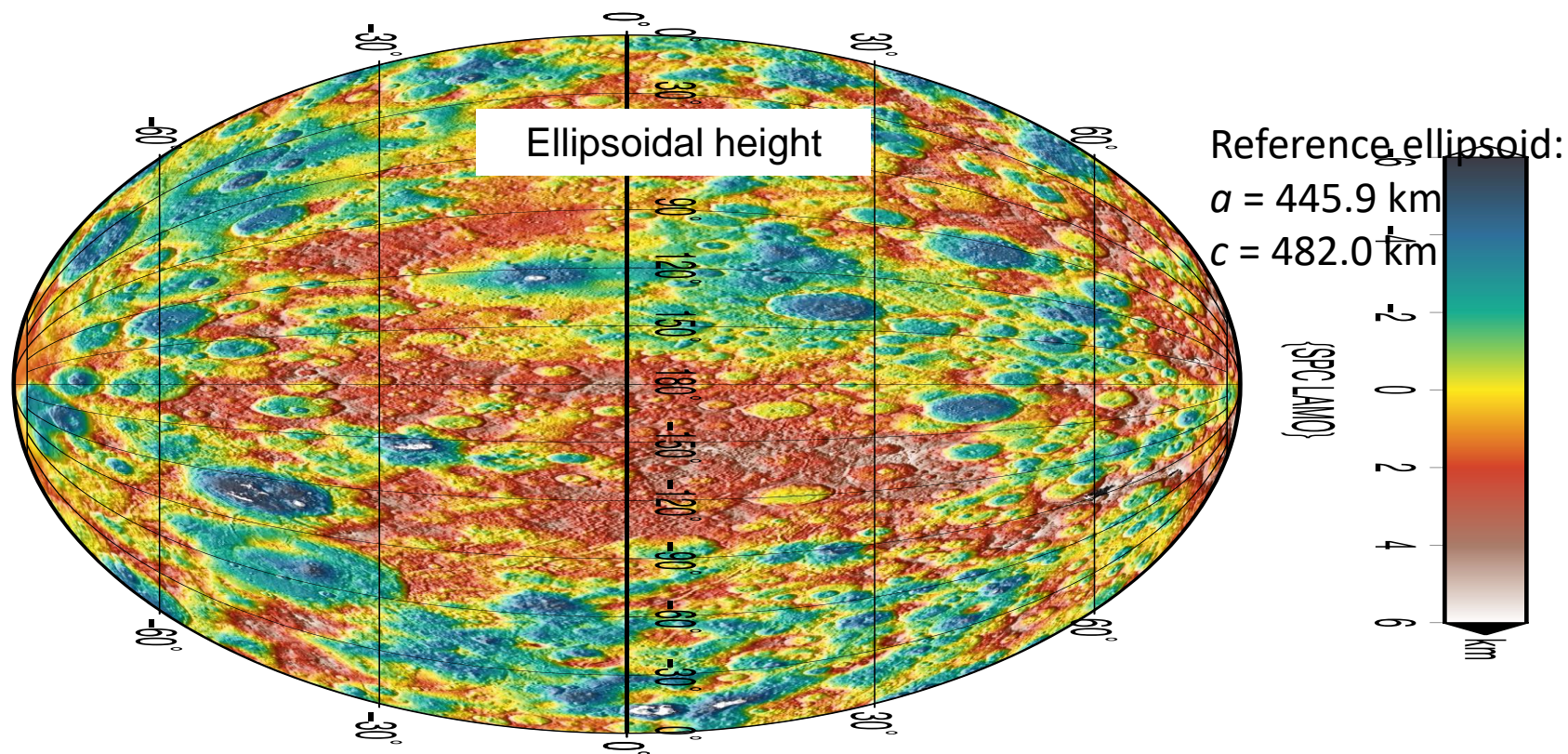


Reference ellipsoid:

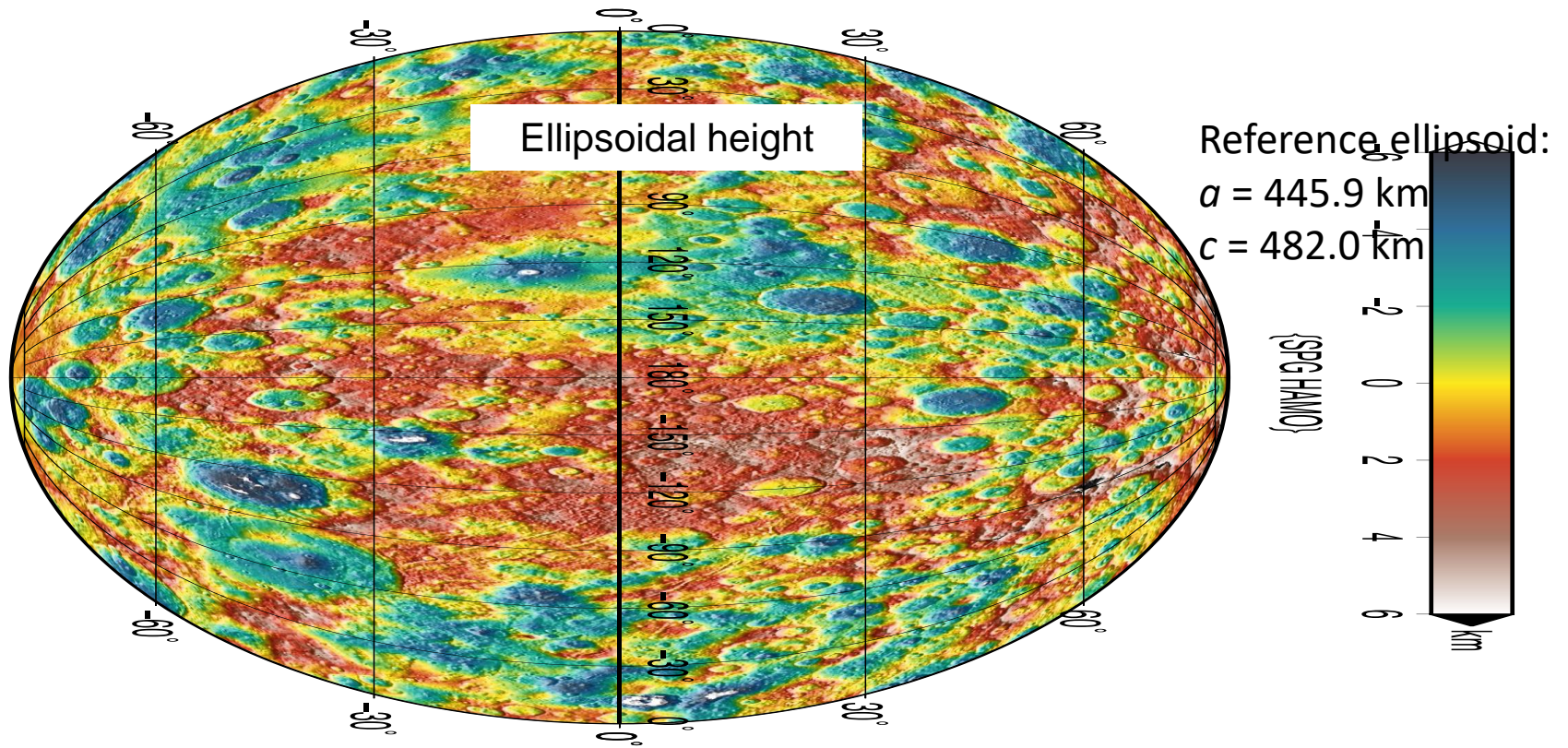
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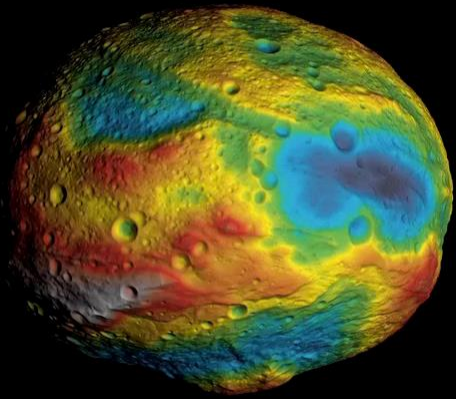
Ceres SPC



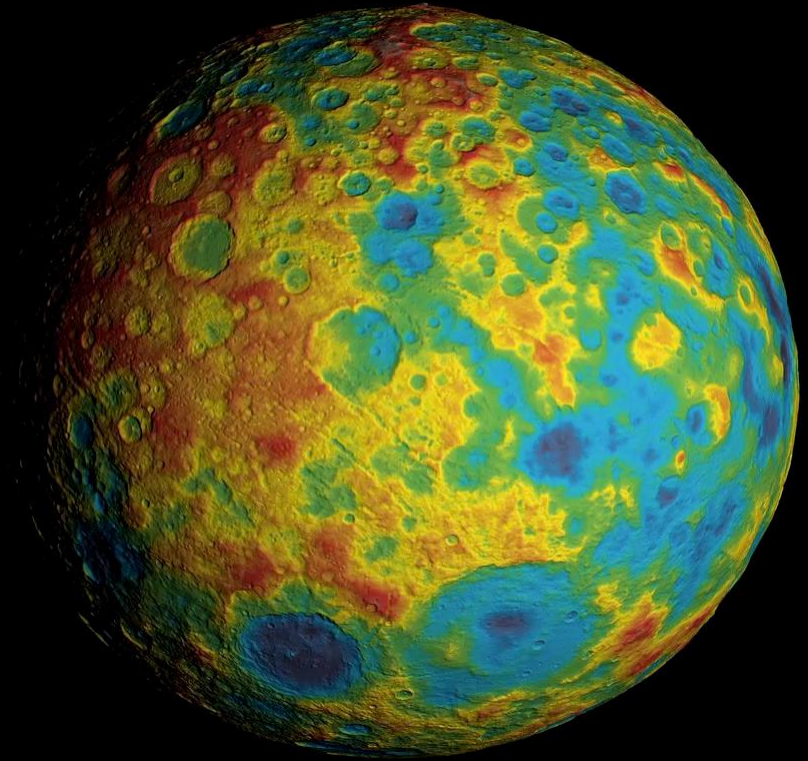
Ceres SPG



Vesta and Ceres

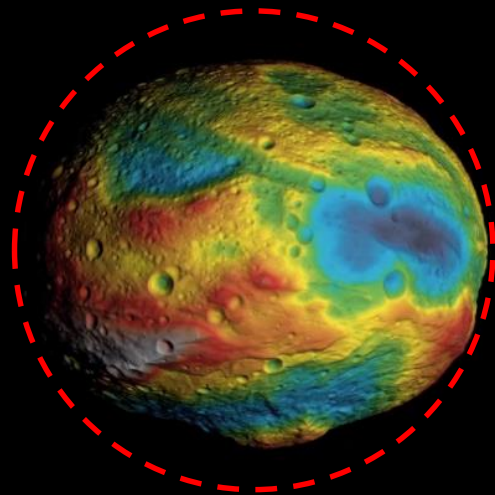


Gaskell, 2012

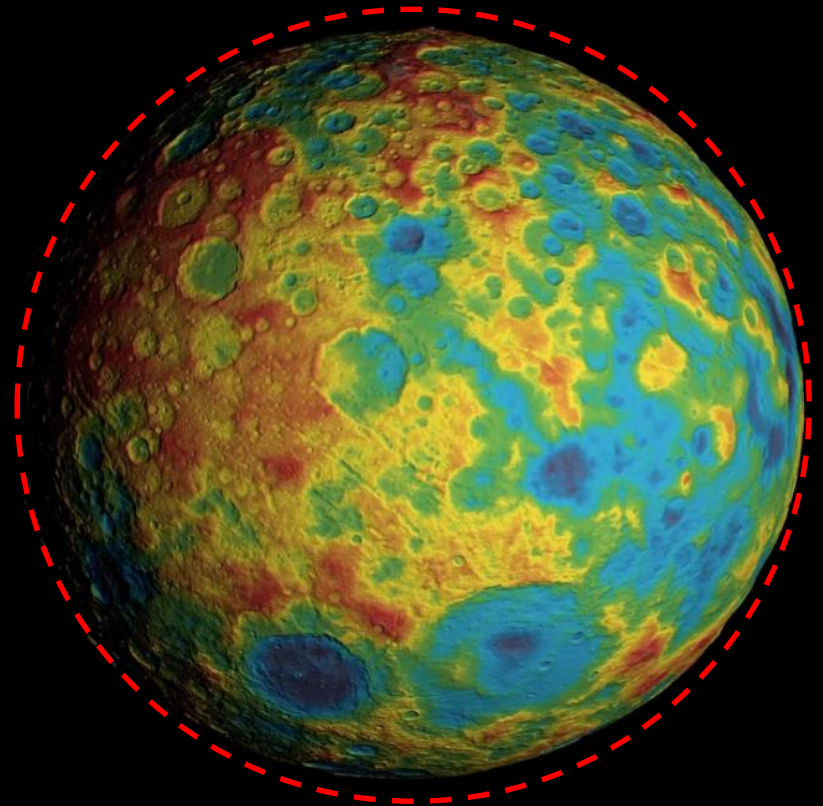


Park et al., 2016

Vesta and Ceres



Gaskell, 2012



Park et al., 2016

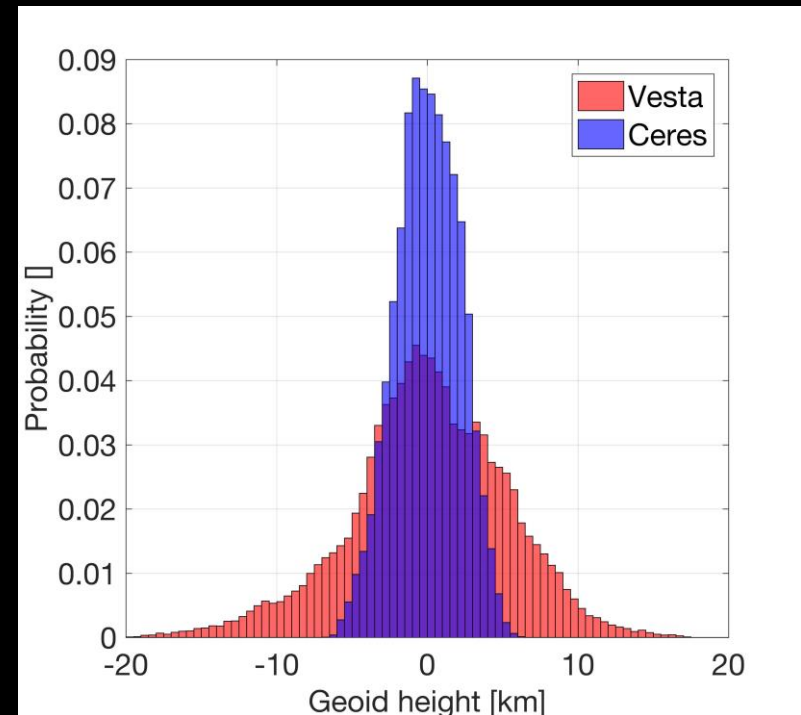
Vesta and Ceres topography

Shape statistics

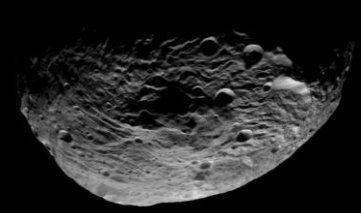
Parameter	Vesta	Ceres
Radius range (km)	80.1	> 44.5
Polar flattening	0.2038	> 0.0770
Equatorial flattening	0.0262	> 0.0043
equatorial/polar	12.9%	> 5.6%
Geoidal height range (km)	37.9	> 13.2
Geoidal height RMS (km)	5.2	> 2.1

- Ceres is closer to hydrostatic equilibrium than Vesta
- Smoother topography at Ceres

Hypsograms of Vesta and Ceres



**Hypsogram* is a fancy word for the “histogram of elevations”

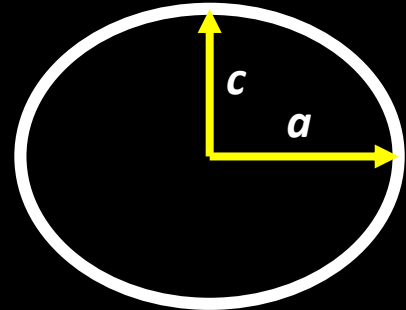


How we use shape data?

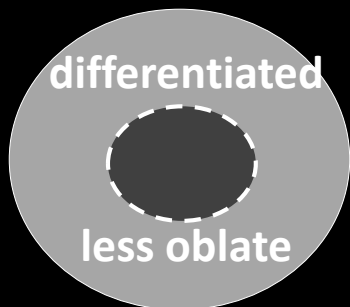


- **Hydrostatic equilibrium**
- **Isostatic compensation**
- **Viscous relaxation**

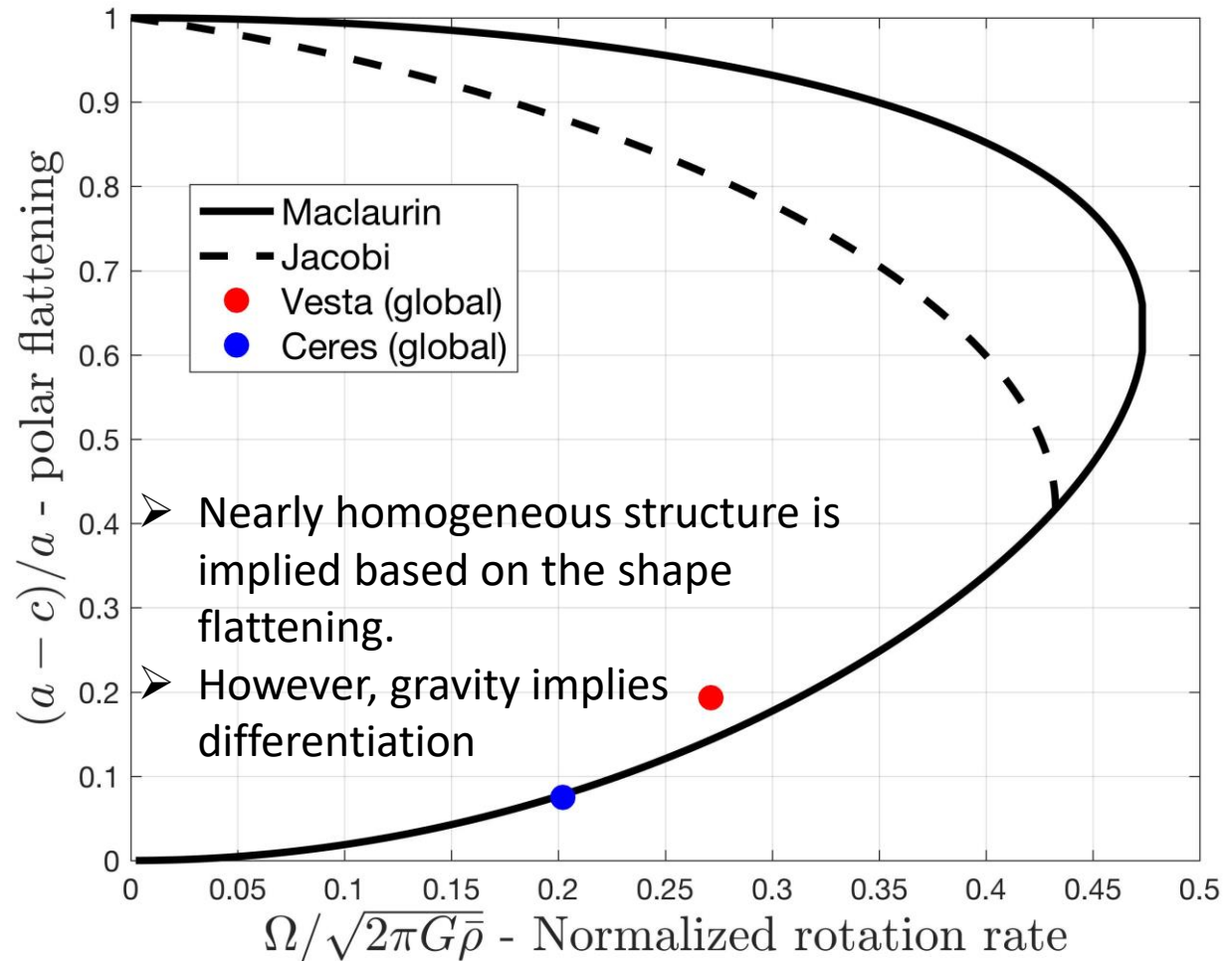
Hydrostatic equilibrium

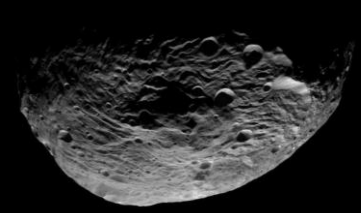


homogeneous
more oblate



differentiated
less oblate





How we use shape data?

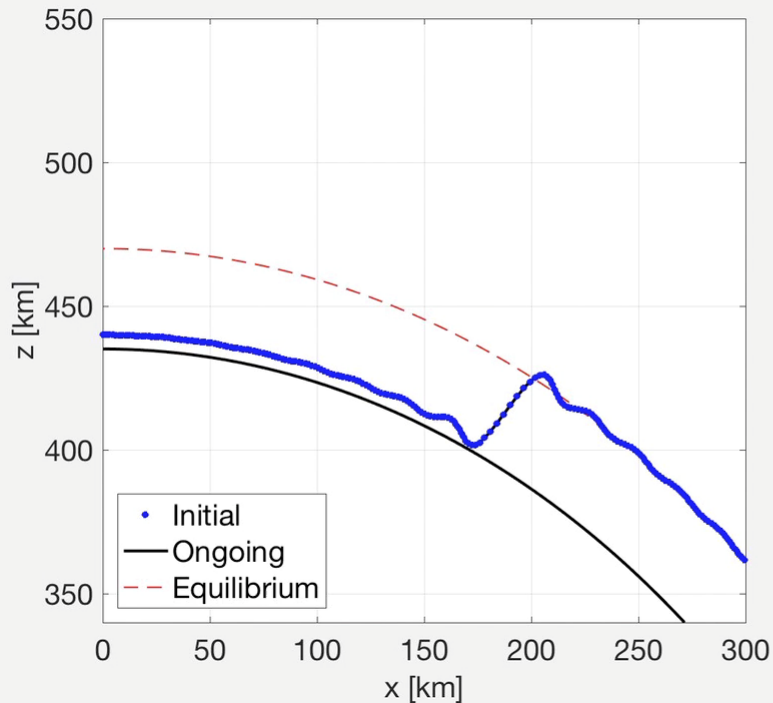


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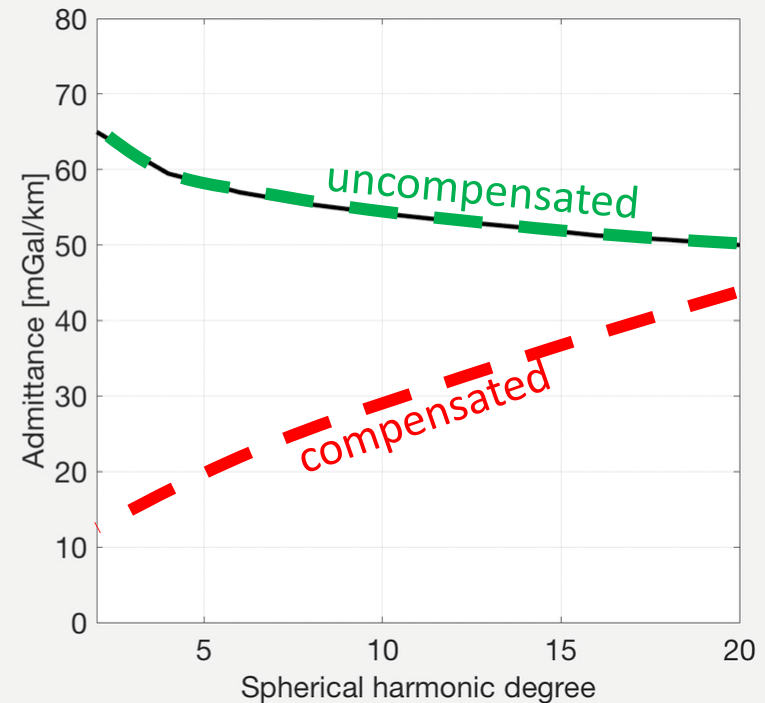
Isostatic compensation

➤ Example of a spherical cap (depression) relaxation

Interface evolution

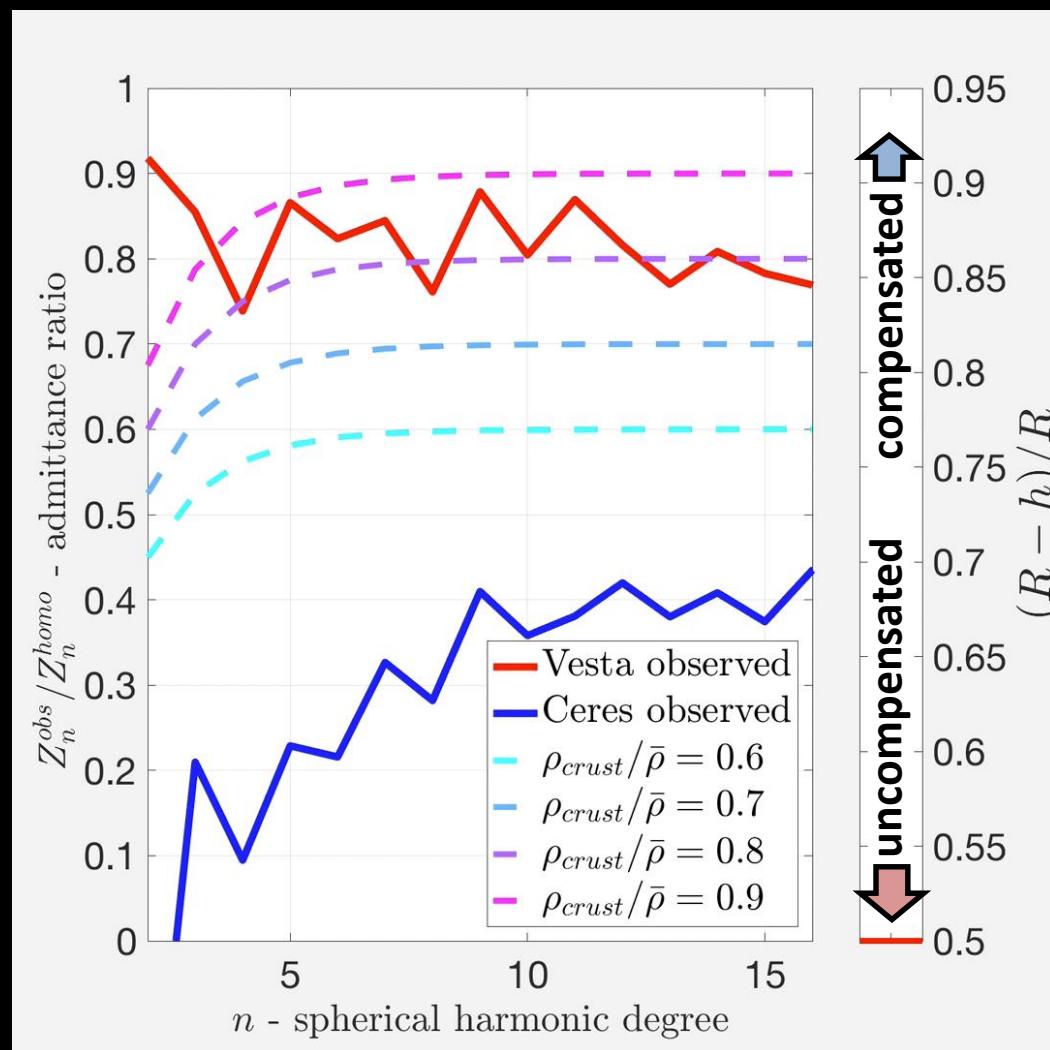


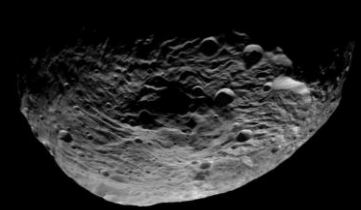
Admittance evolution
= ratio of gravity to topography



Isostatic compensation

- Admittance (Z) is a ratio of gravity to topography.
- Isostatically compensated and uncompensated topography have different admittances.
- Modeling of isostasy allows constraining the density and thickness of the compensated layer as well as the density contrast.

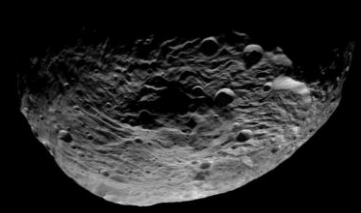




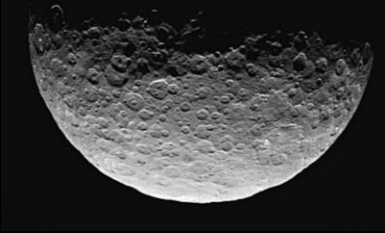
Compensation for **Vesta** and **Ceres**



- **Vesta** topography is uncompensated
- **Vesta** acquired most of its topography when the crust was already cool and not-relaxing
- **Ceres** topography is compensated
- Lower viscosities (compared to Vesta) enabled relaxation of topography to the isostatic state



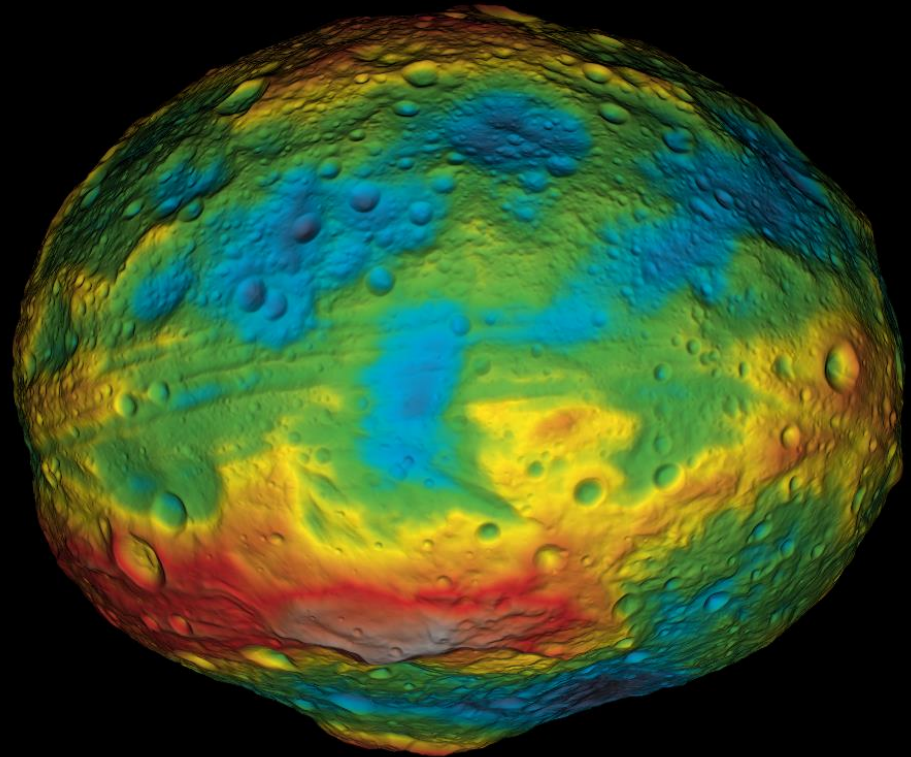
How do we use shape data?



- **Hydrostatic equilibrium**
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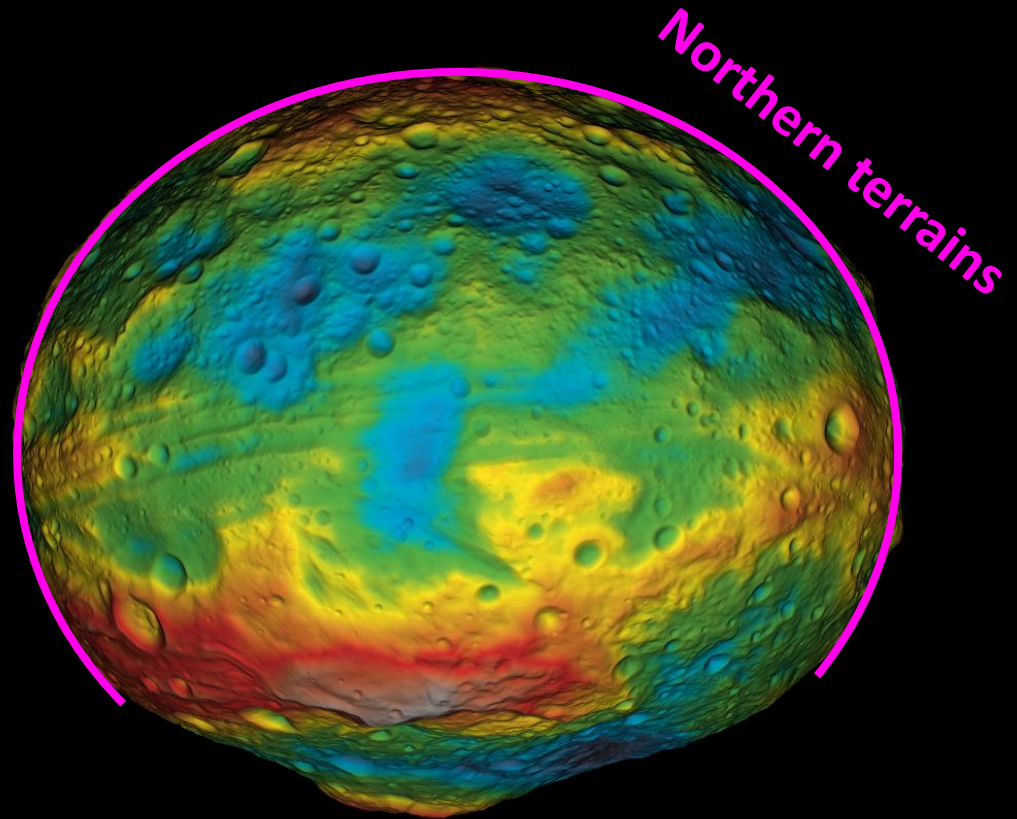
Early efficient viscous relaxation of Vesta

- Vesta was likely close to hydrostatic equilibrium in its early history (Fu et al., 2014).
- Vesta's northern terrains likely reflect its pre-impact equilibrium shape.
- Major impact occurred when Vesta was effectively non-relaxing leading to uncompensated Rheasilvia and Veneneia basins.



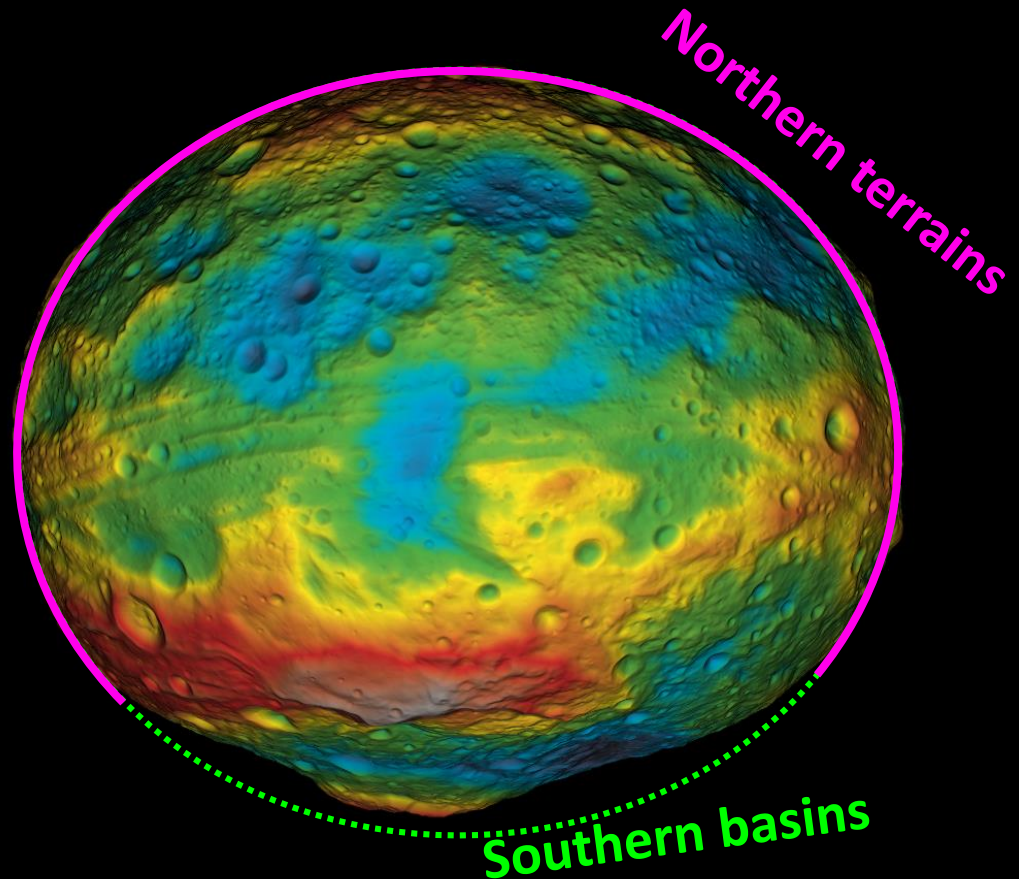
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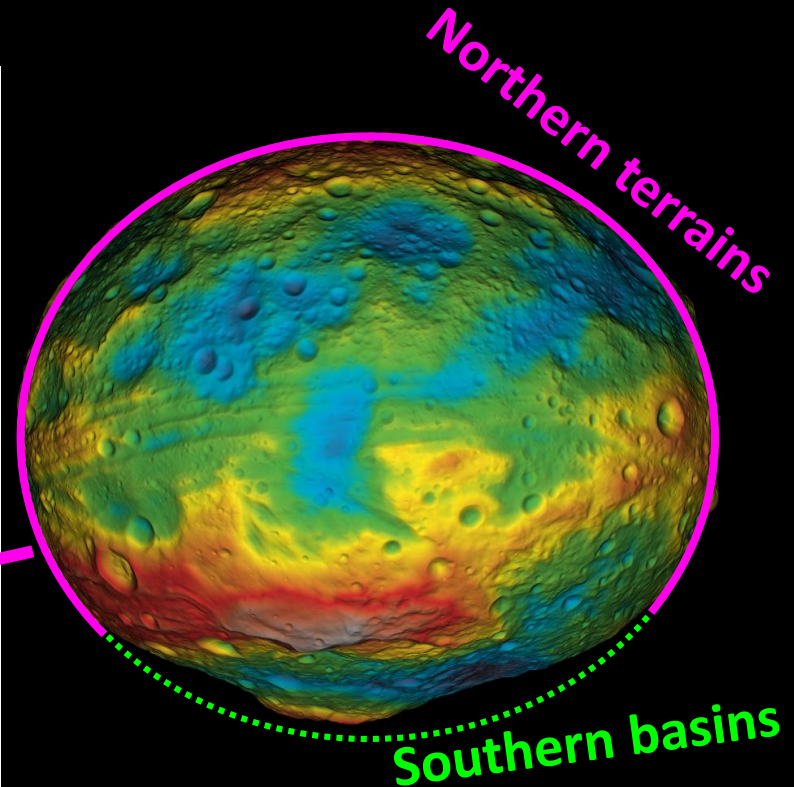
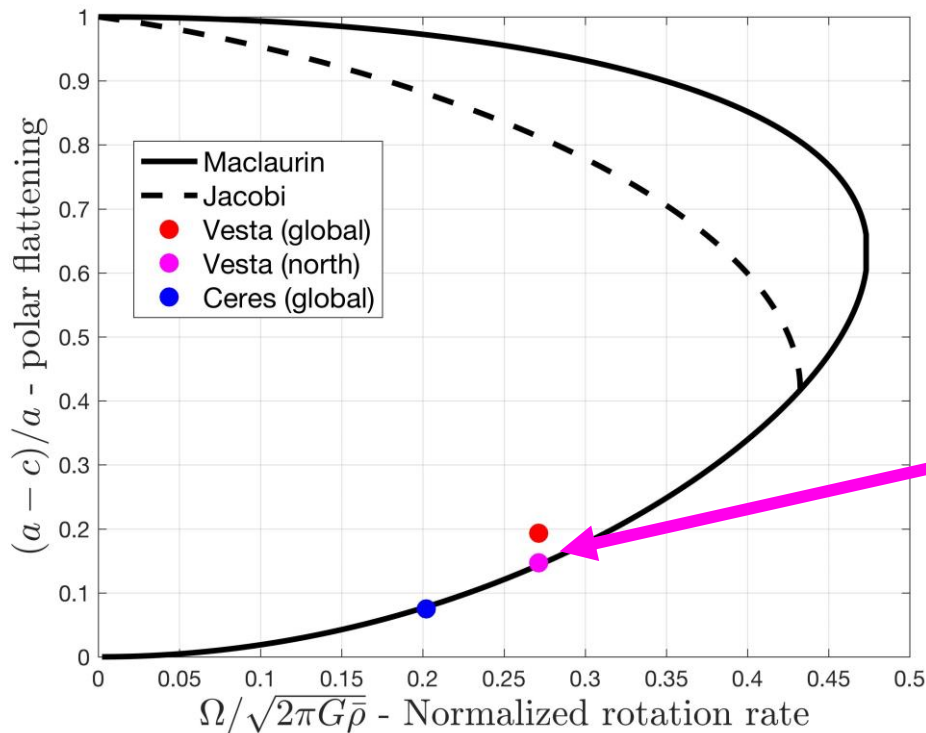


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Early efficient viscous relaxation of Vesta





Viscous relaxation on Ceres

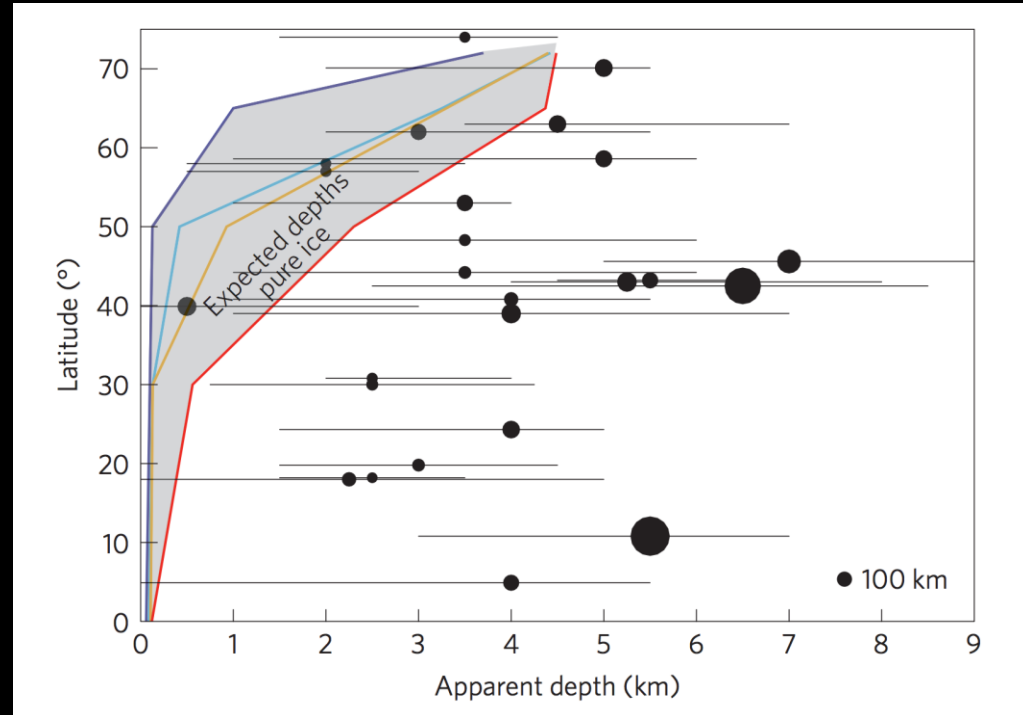


- Bland et al., 2013 predicted that craters on Ceres would quickly relax in an ice-dominated shell
 - Equatorial warmer craters would relax faster than colder polar craters
- Bland et al., 2016 did not find that evidence for such relaxation pattern
 - No latitude dependence of crater depth

Viscous relaxation on Ceres

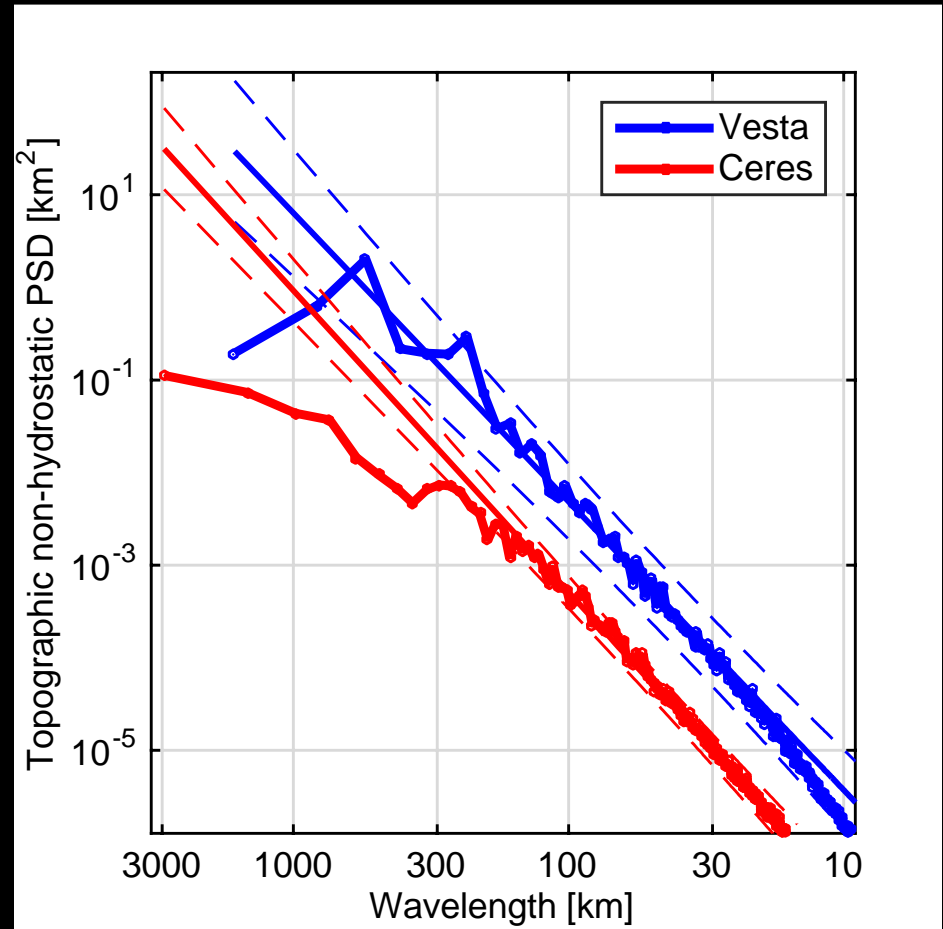
- Bland et al., 2013 predicted that craters on Ceres would quickly relax in an ice-dominated shell
 - Equatorial warmer craters would relax faster than colder polar craters
- Bland et al., 2016 did not find evidence for such relaxation pattern
 - No latitude dependence of crater depth

Crater depth study



Evidence for viscous relaxation

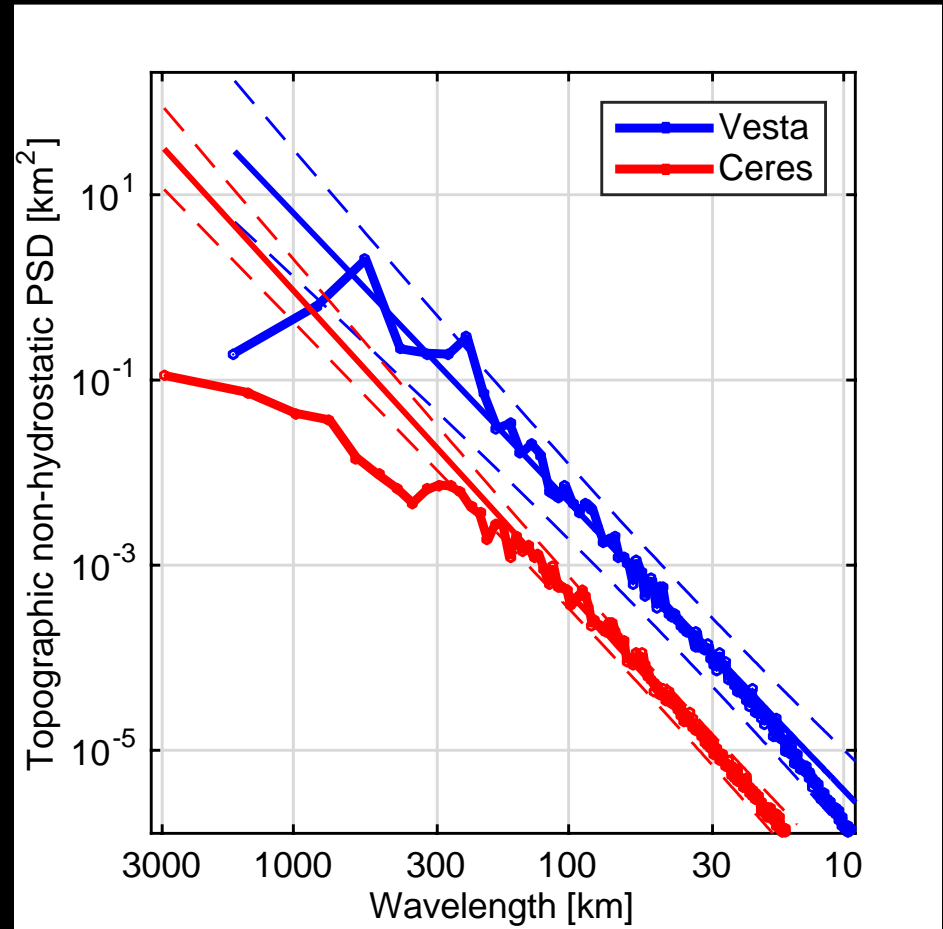
- **More general approach: study topography power spectrum**
- **Power spectra for Vesta closely fits with the power law to the lowest degrees ($\lambda < 750$ km)**
- **Ceres power spectrum deviates from the power law at $\lambda > 270$ km**



Ermakov et al., 2017

Evidence for viscous relaxation

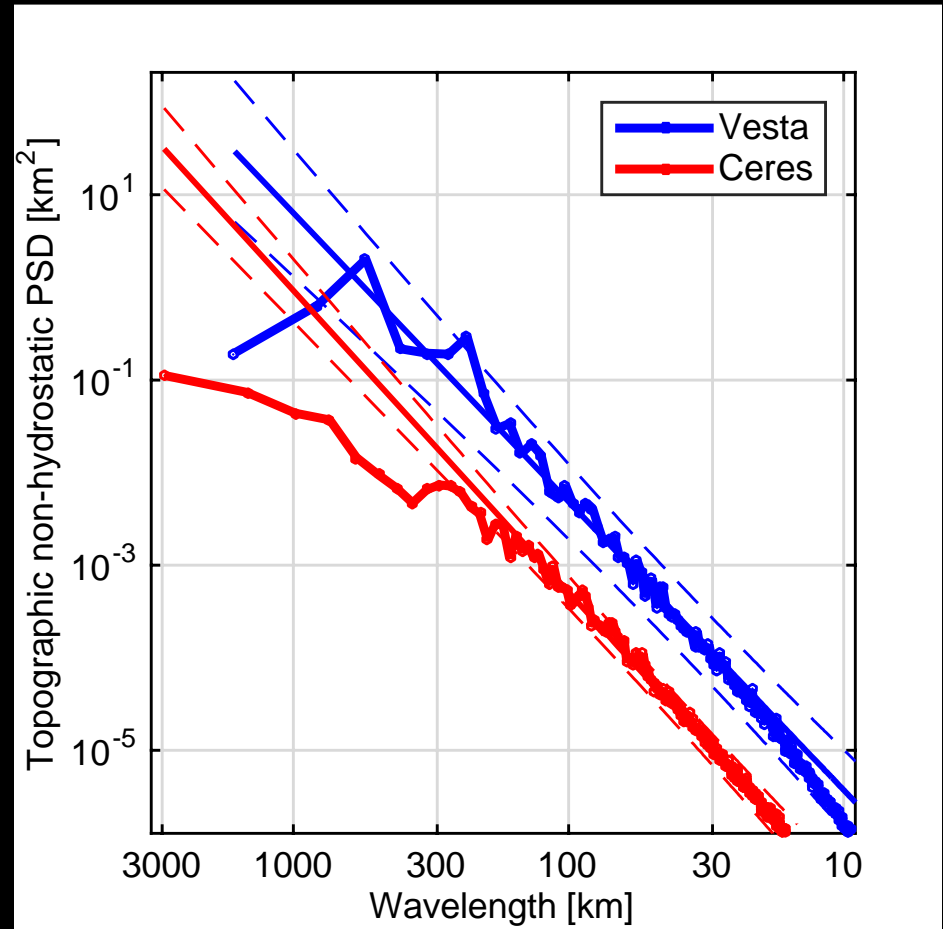
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Ermakov et al., 2017

Evidence for viscous relaxation

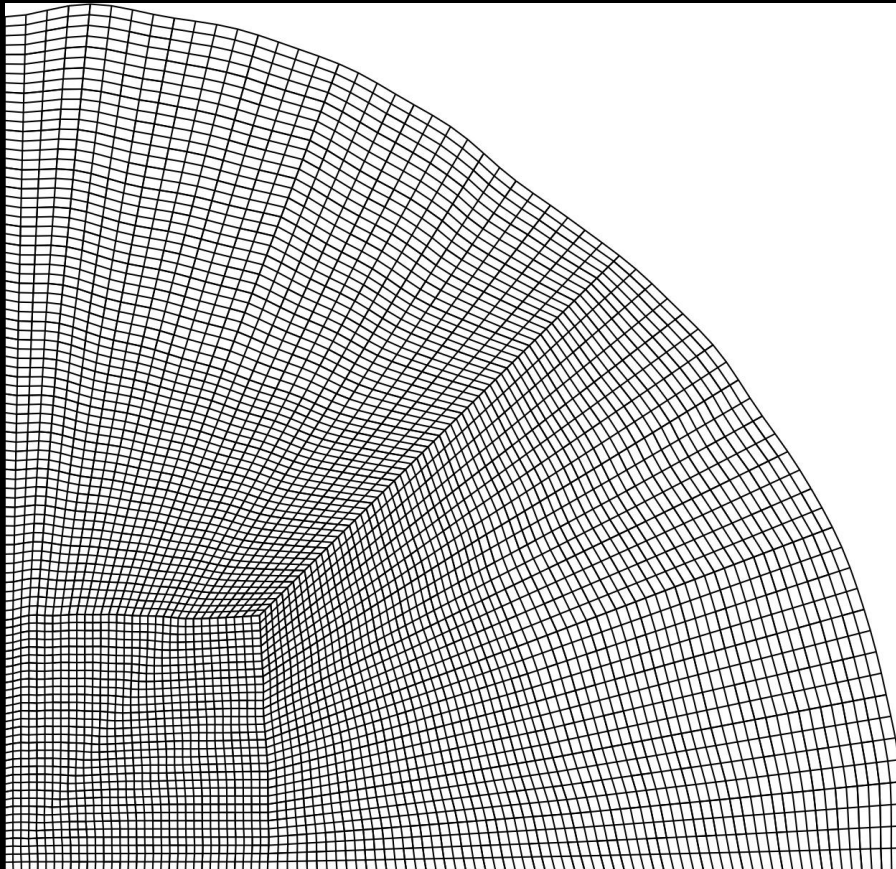
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Ermakov et al., 2017



Finite element model

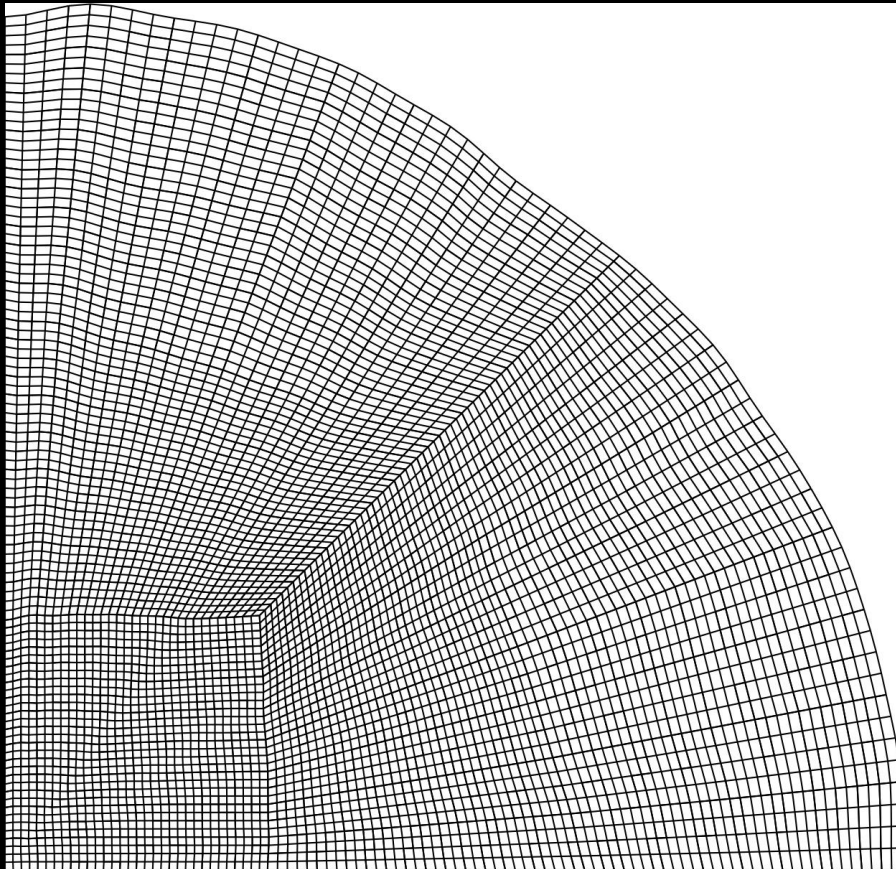


- Assume a density and rheology structure
- Solve Stokes equation for an incompressible flow using deal.ii library
- Compute the evolution of the outer surface power spectrum

Fu et al., 2014; Fu et al, 2017



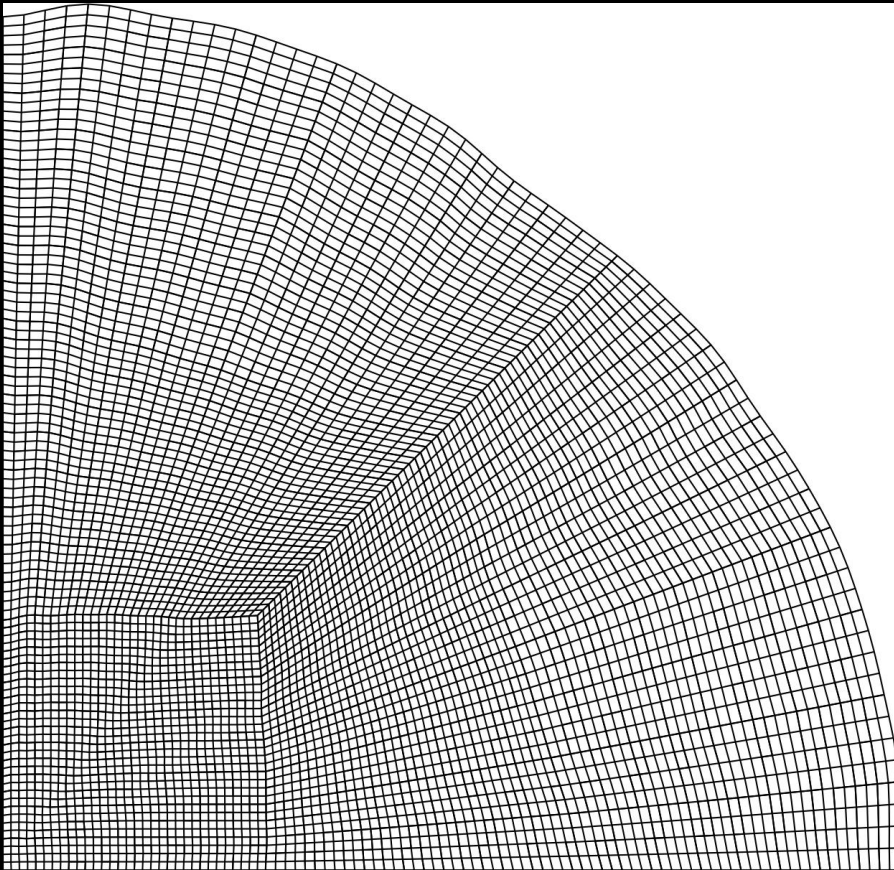
Finite element model



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Finite element model

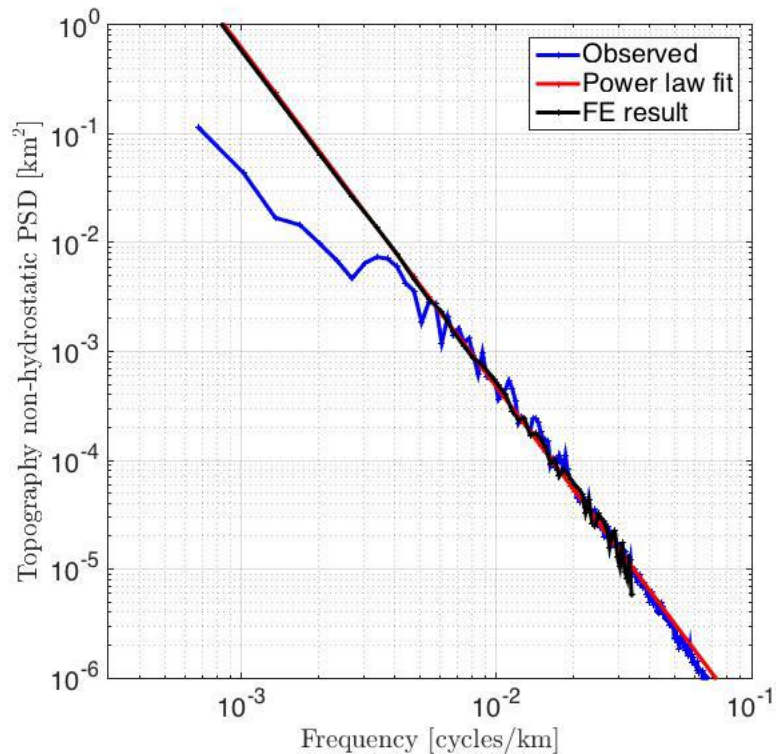


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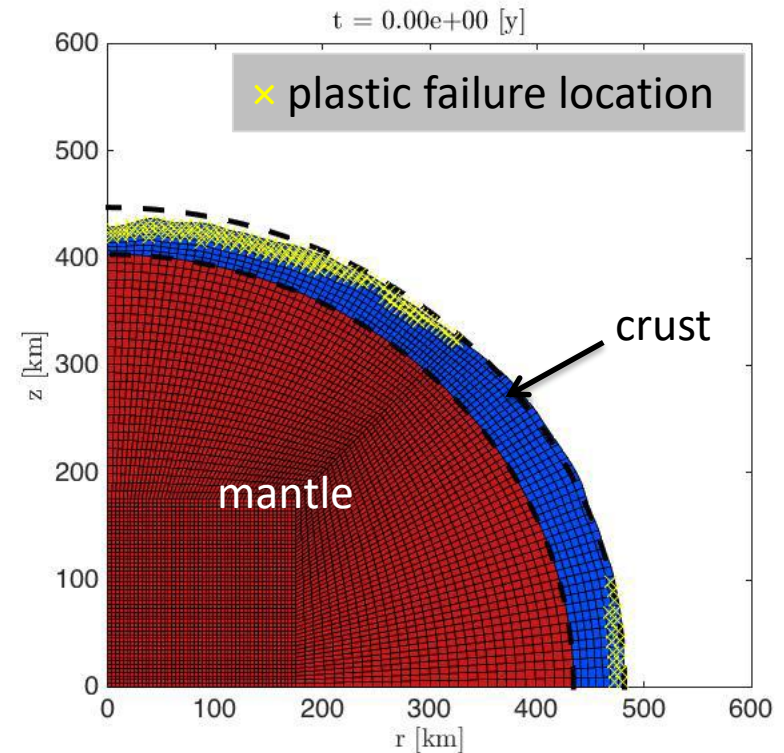
Fu et al., 2014; Fu et al, 2017

Example of a FE modeling run

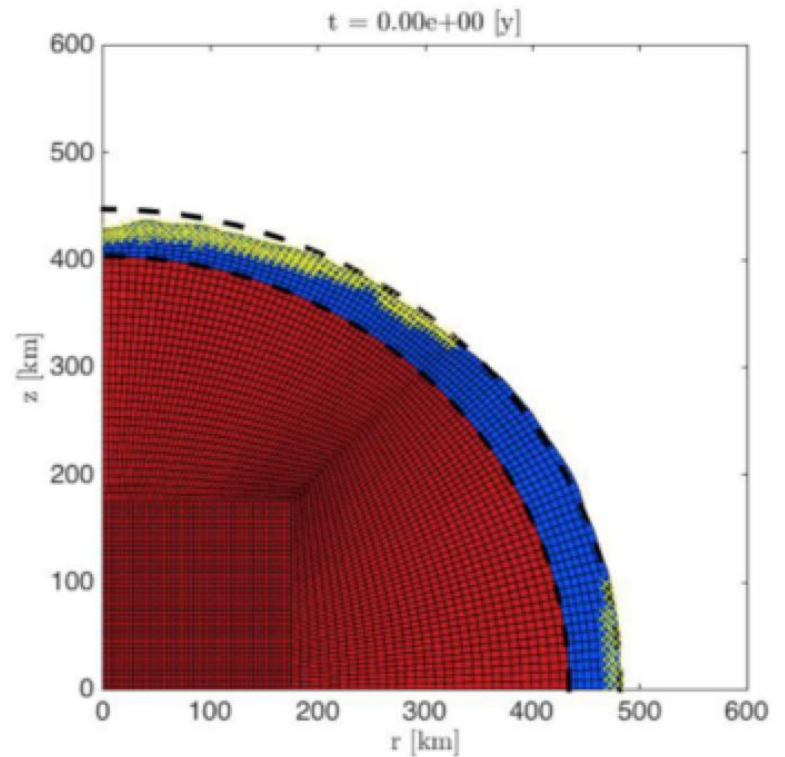
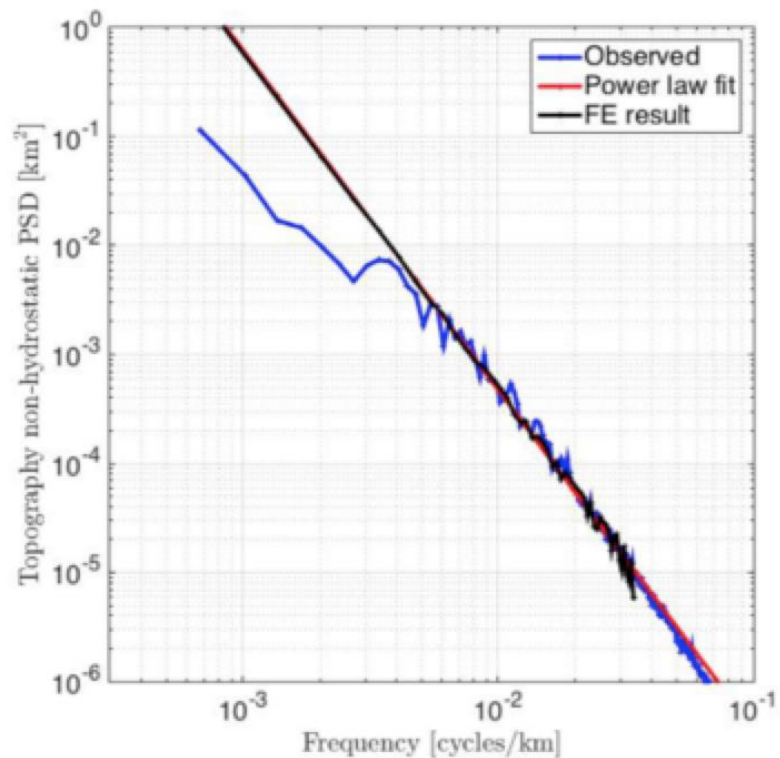
relaxation in the frequency domain



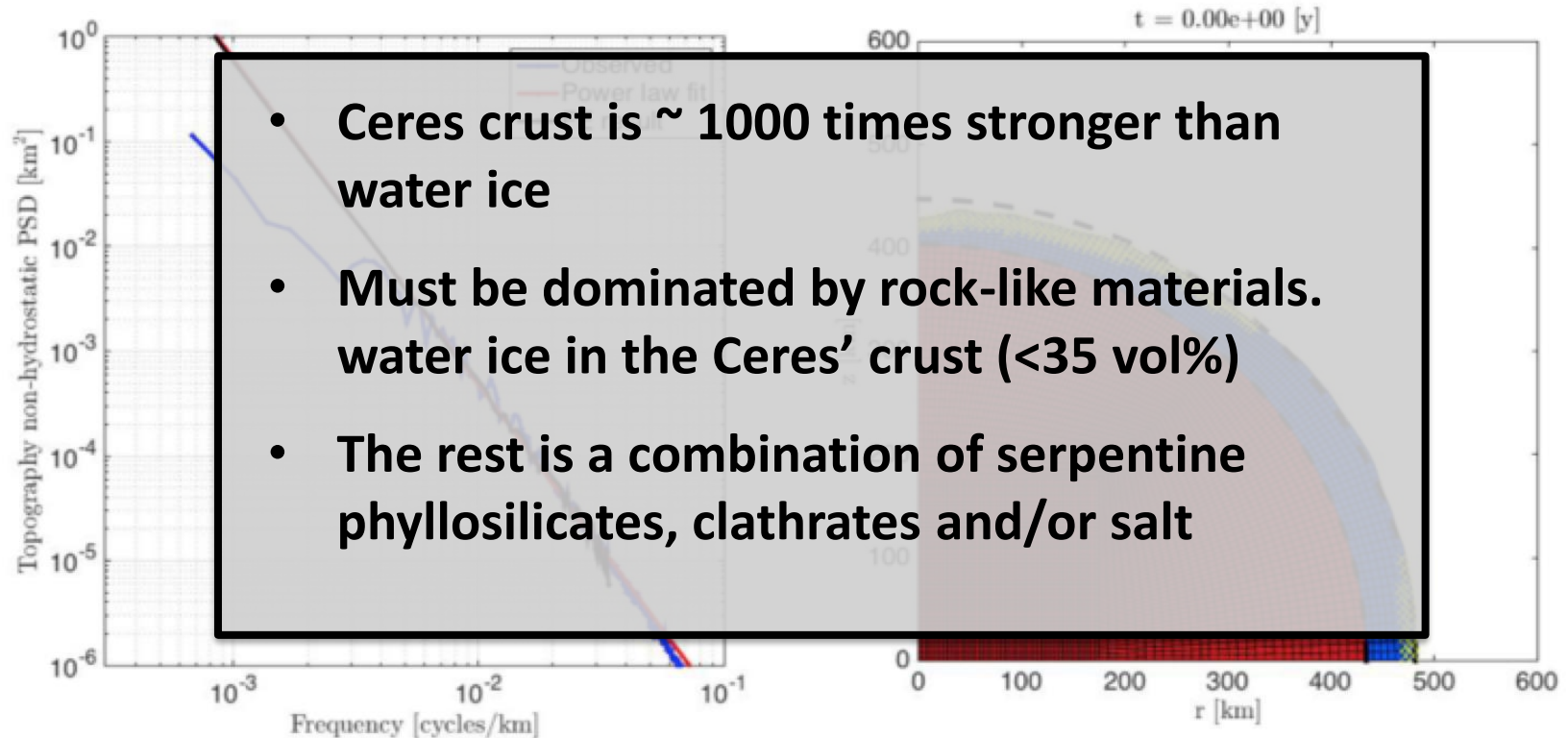
relaxation in the spatial domain

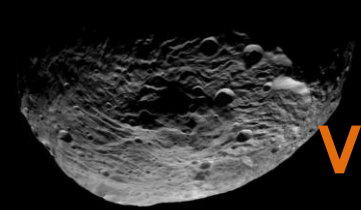


Finite element modeling results



Finite element modeling results

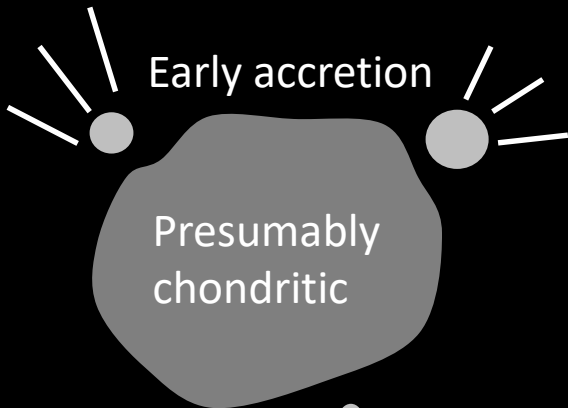




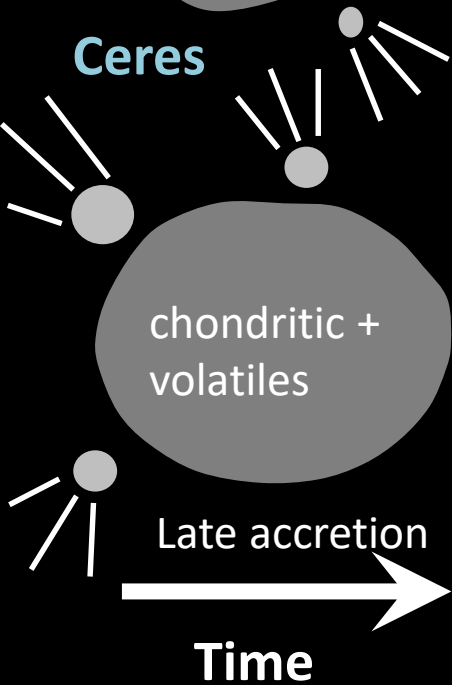
Vesta and Ceres comparative evolution



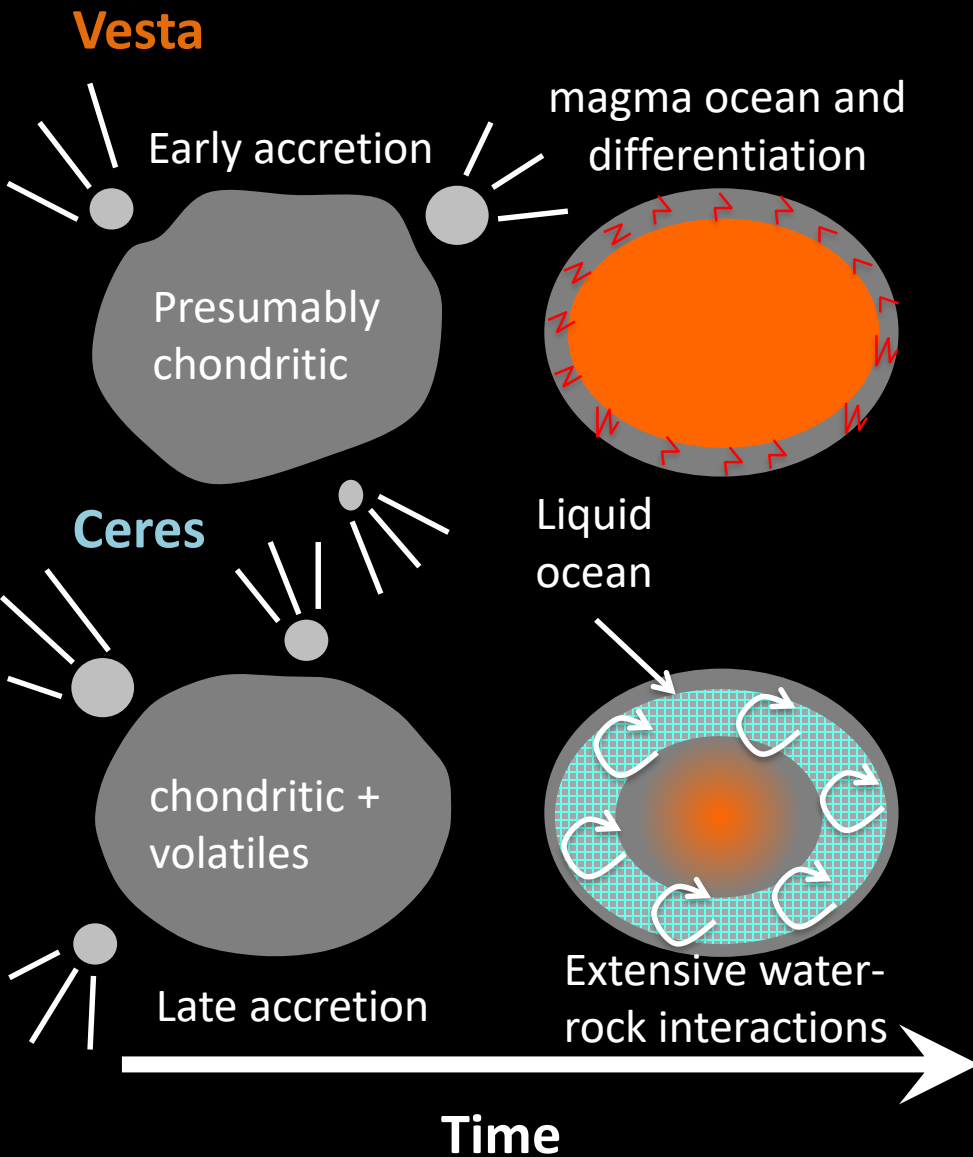
Vesta



Ceres

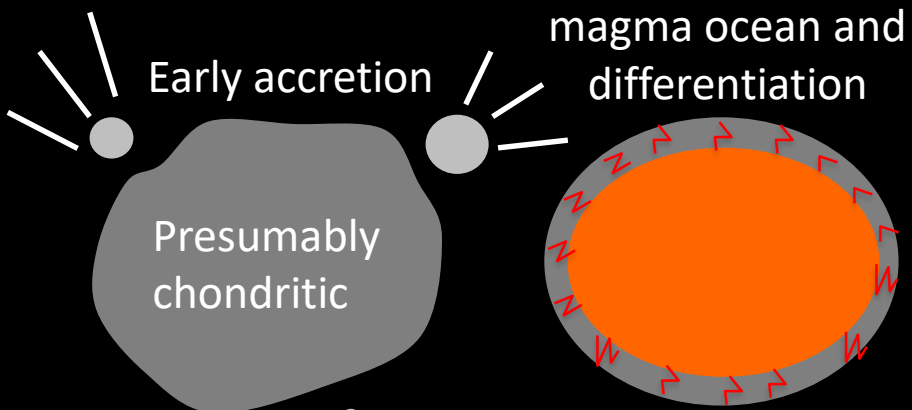


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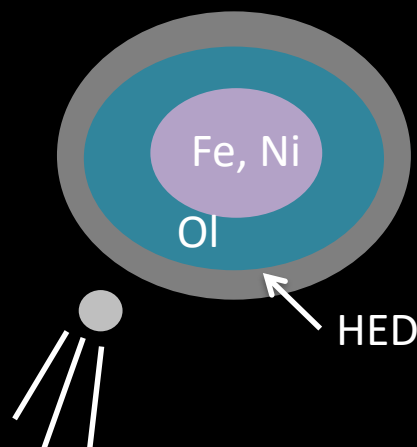


Vesta and Ceres comparative evolution

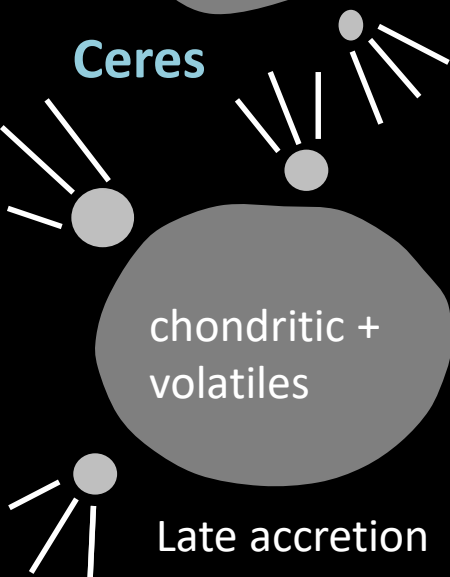
Vesta



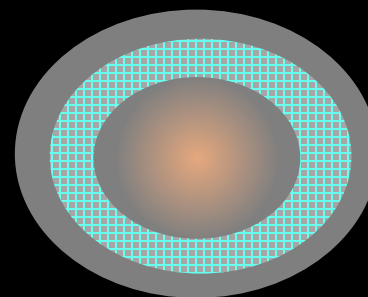
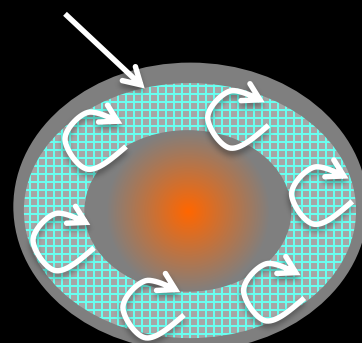
giant impact into cool Vesta



Ceres



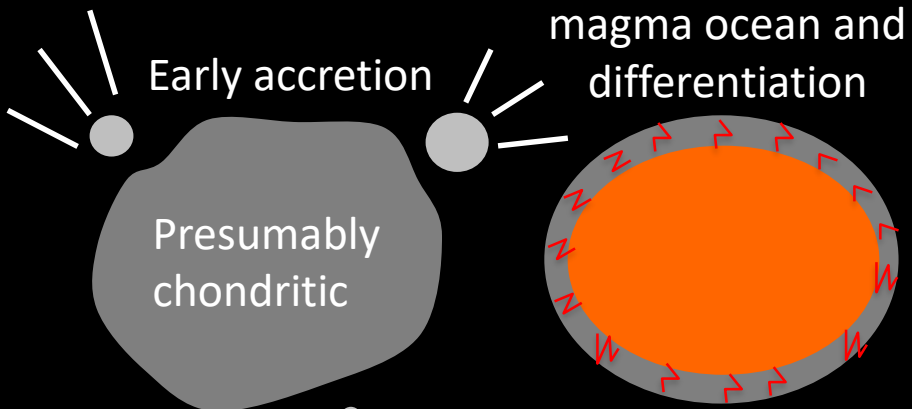
Liquid ocean



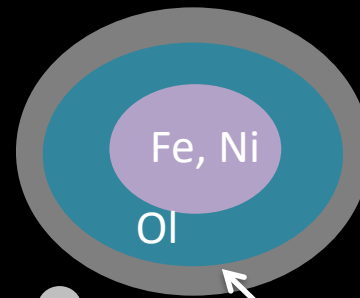
Time

Vesta and Ceres comparative evolution

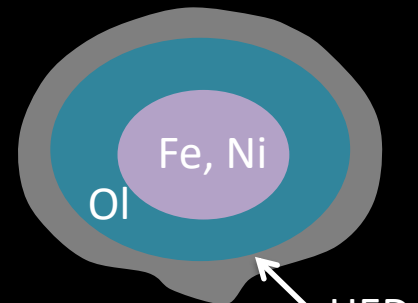
Vesta



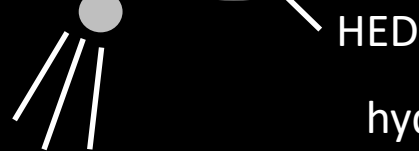
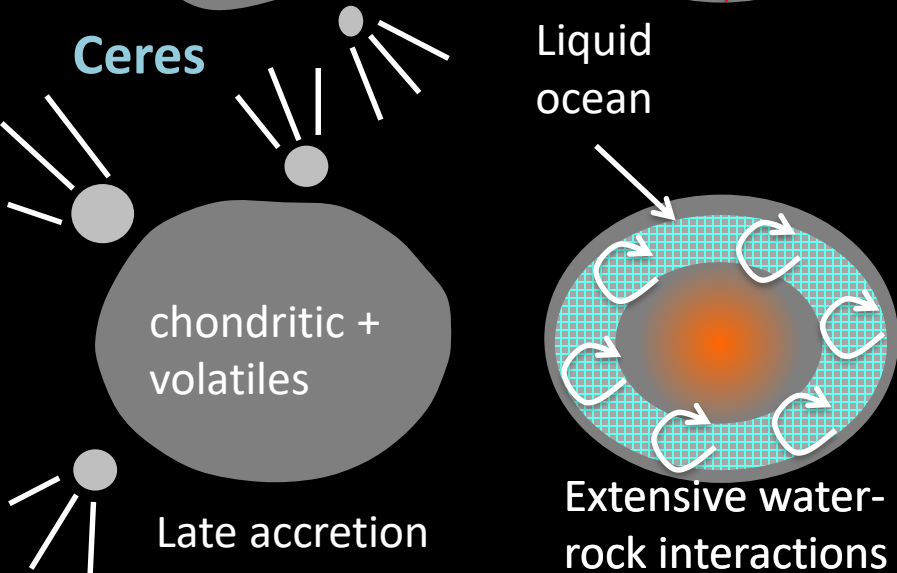
giant impact into cool Vesta



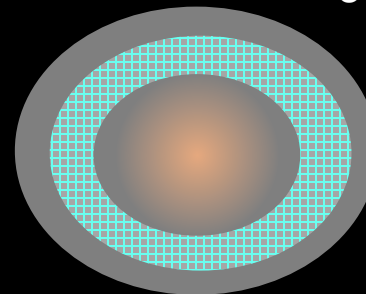
Present-state



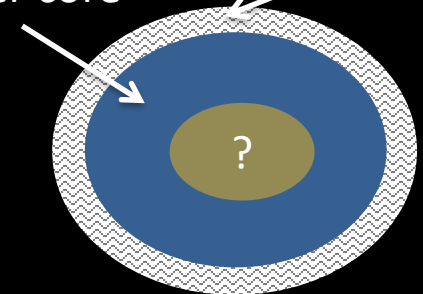
Ceres



hydrated outer core



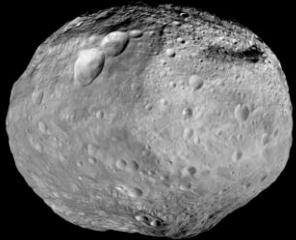
hydrated salts water ice, rock



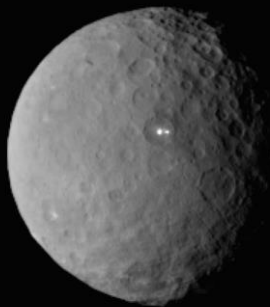
Present-state

Time

Summary



- Formed early (< 5 My after CAI)
- Once hot and hydrostatic, **Vesta** is no longer either
- Differentiated interior
- Most of topography acquired when **Vesta** was already cool \Rightarrow uncompensated topography
- Combination of gravity/topography data with meteoritic geochemistry data provides constraints on the internal structure

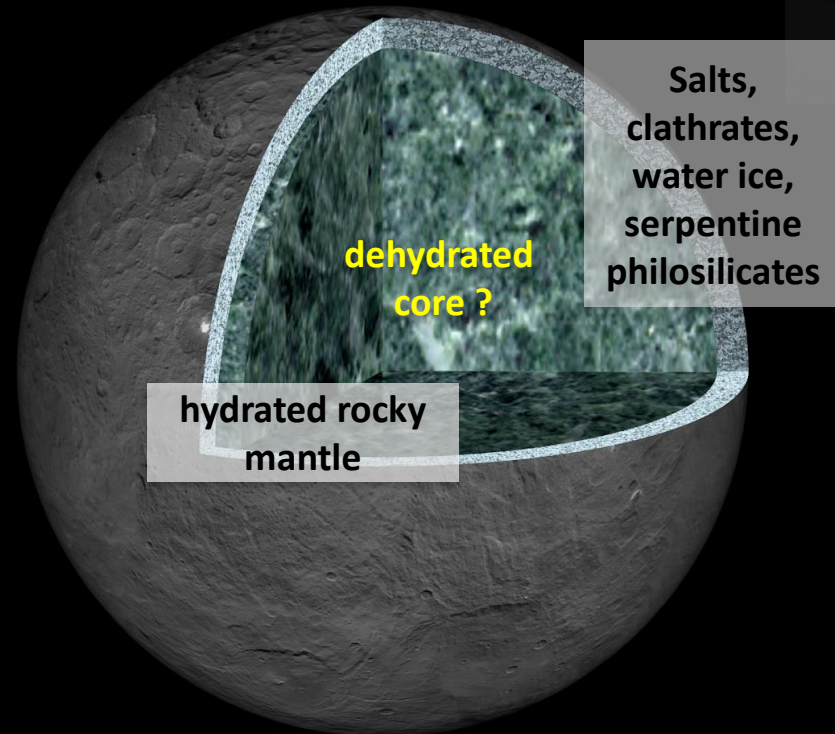


- Cooler history
 - late formation
 - and/or heat transfer due to hydrothermal circulation
- Partially differentiated interior
- Experienced viscous relaxation
- Much lower surface viscosities (compared to Vesta) allowed compensated topography
- **Ceres'** crust is light (based on admittance analysis) and strong (based on FE relaxation modeling)
- Not much water ice in **Ceres** crust (< 35 vol%) now

Internal structures of Vesta and Ceres

Ceres →

- Crust is light (1.1-1.4 g/cc) and mechanically rock-like w
- Mantle density ~2.4 g/cc and unlithified at least to a depth of 100 km
- Possible dehydrated rocky core remains unconstrained



← Vesta

- Crustal density constrained by HEDs and admittance (2.8 g/cc)
- Assuming density of iron meteorites (5-8 g/cc), the core radius is 110 – 155 km

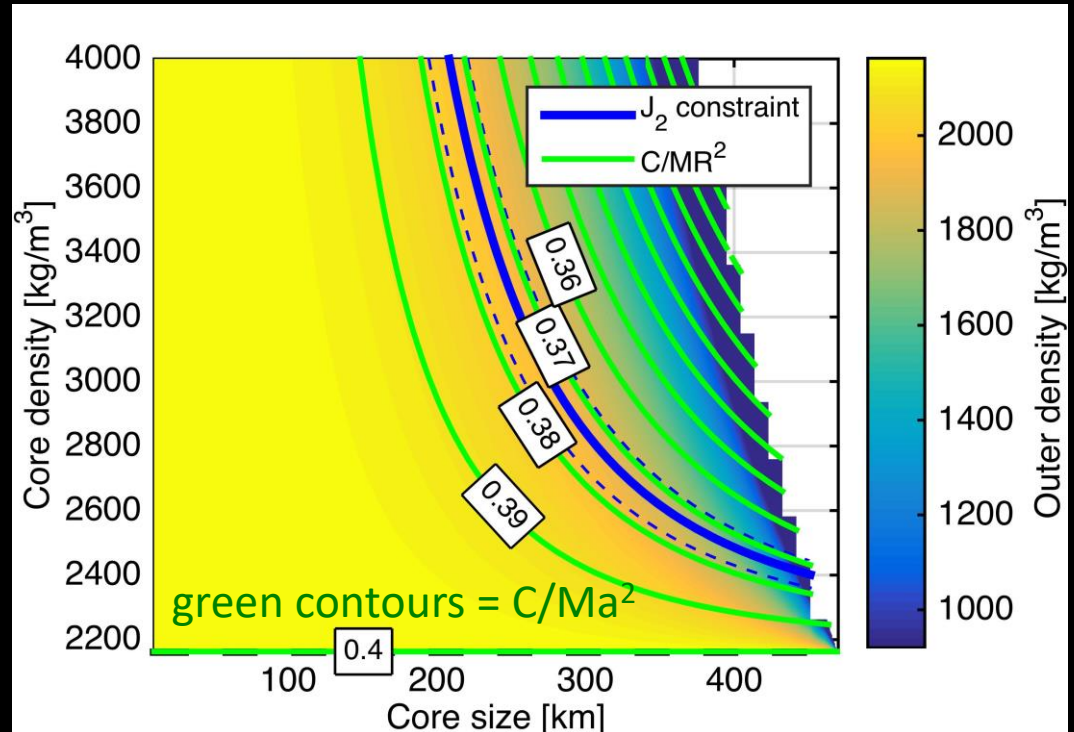


Backup slides



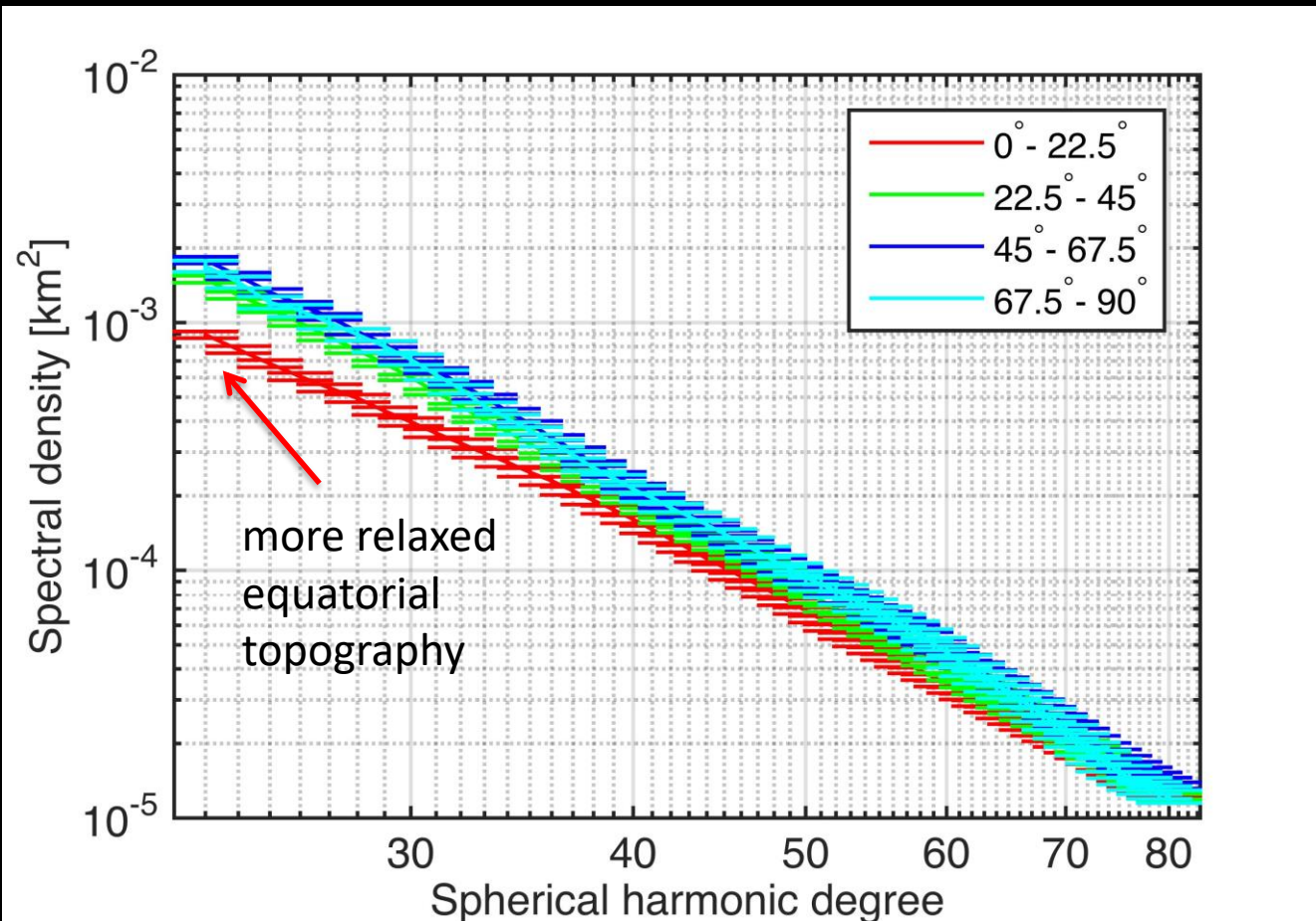
Two-layer model

- Simplest model to interpret the gravity-topography data
- Only 5 parameters: two densities, two radii and rotation rate
- Yields $C/Ma^2 = 0.373$
 $C/M(R_{\text{vol}})^2 = 0.392$



Using Tricarico 2014 for computing hydrostatic equilibrium

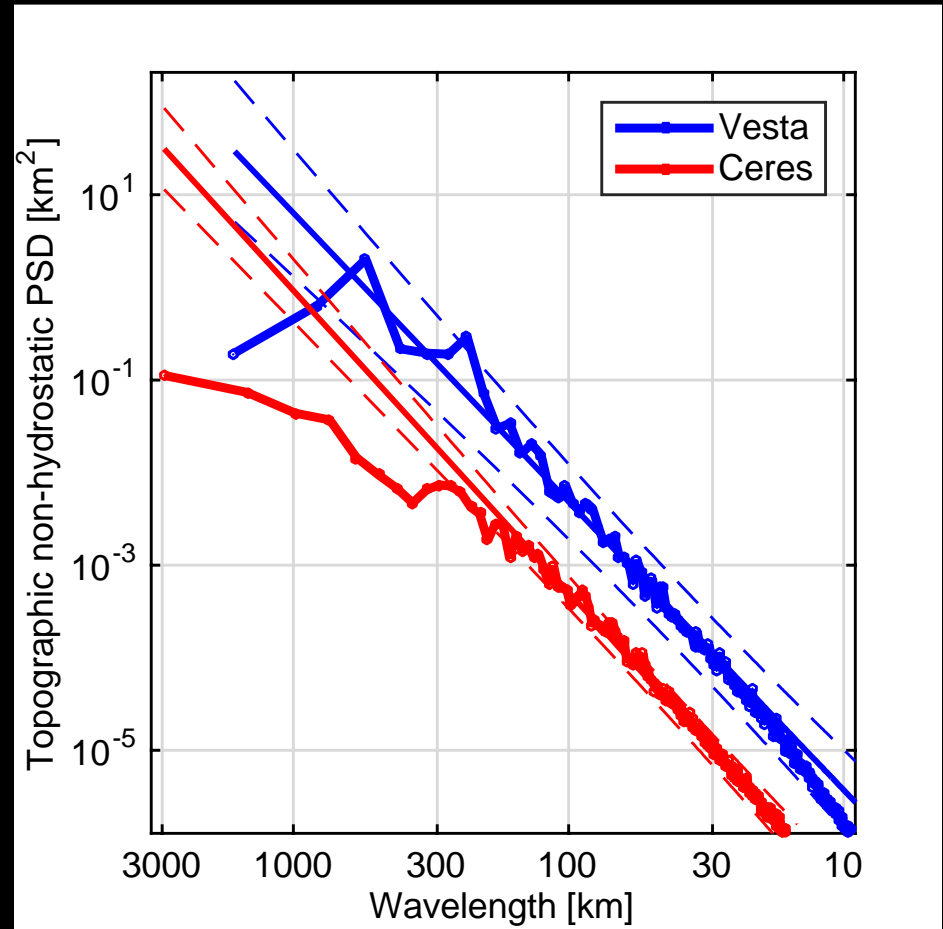
Latitude dependence of relaxation



Ermakov et al., in prep

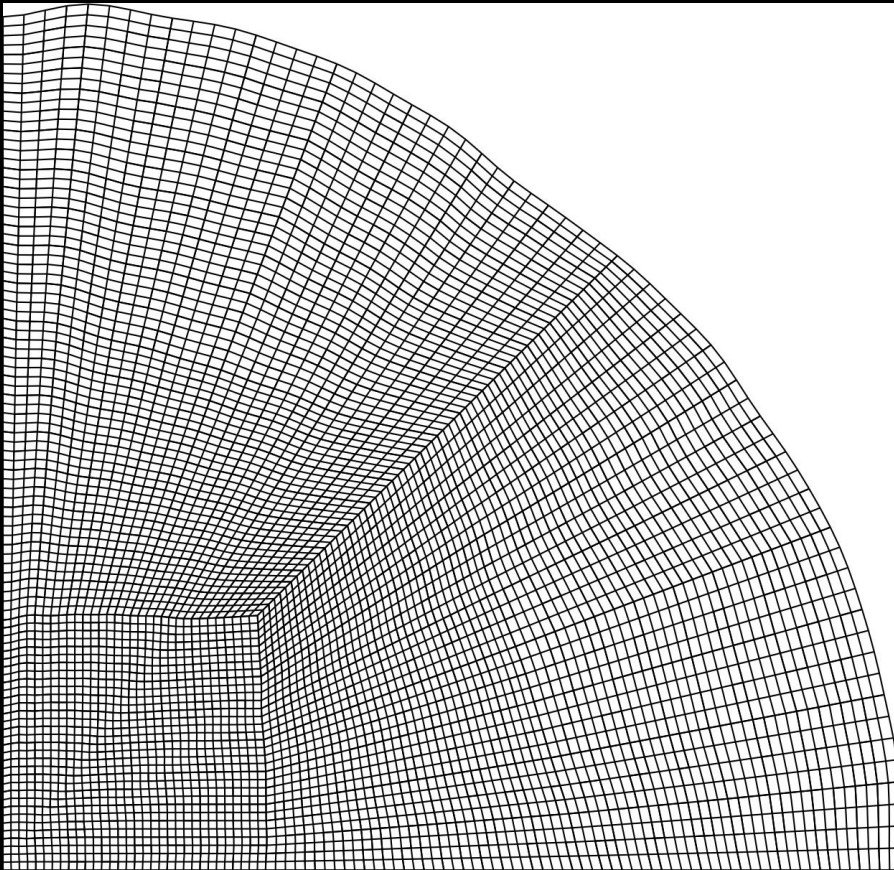
Evidence for viscous relaxation

- More general approach: study topography power spectrum
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Ermakov et al., in prep

Finite element model



Fu et al., 2014; Fu et al,
submitted to EPSL

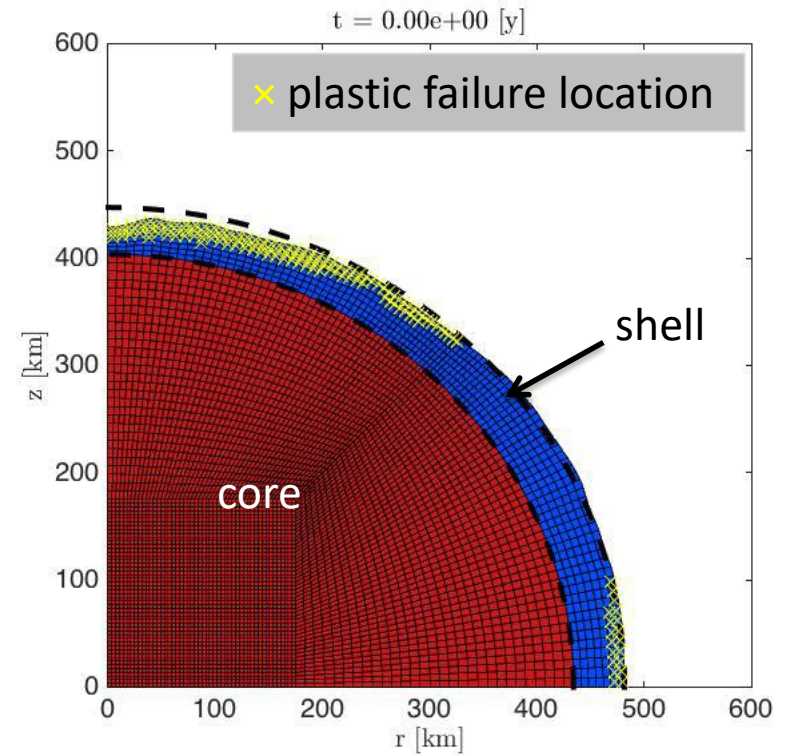
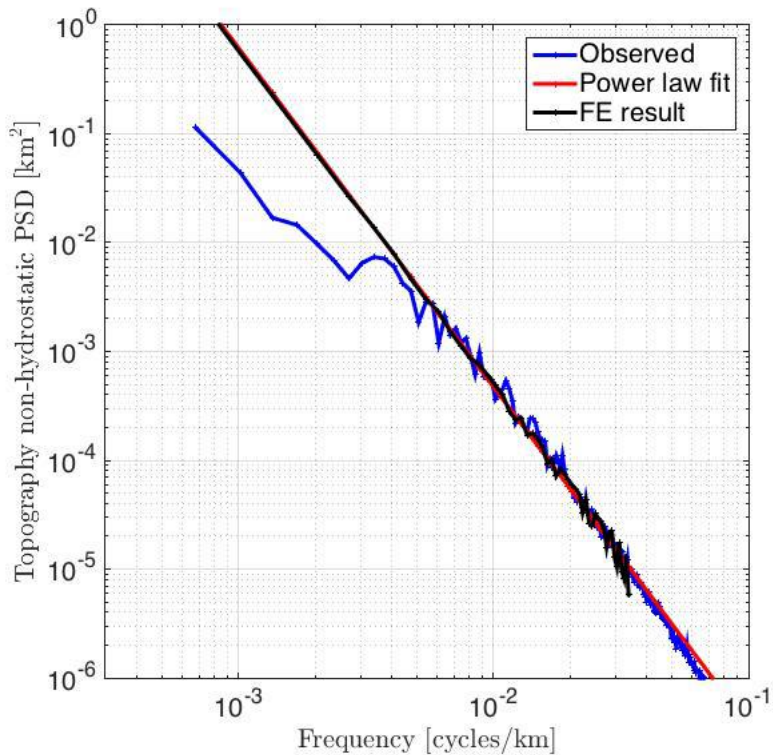
- Assume a density and rheology structure
- Solve Stokes equation for an incompressible flow using deal.ii library

$$\partial_i (2\eta \dot{\epsilon}_{ij}) - \partial_i p = -g_i \rho$$

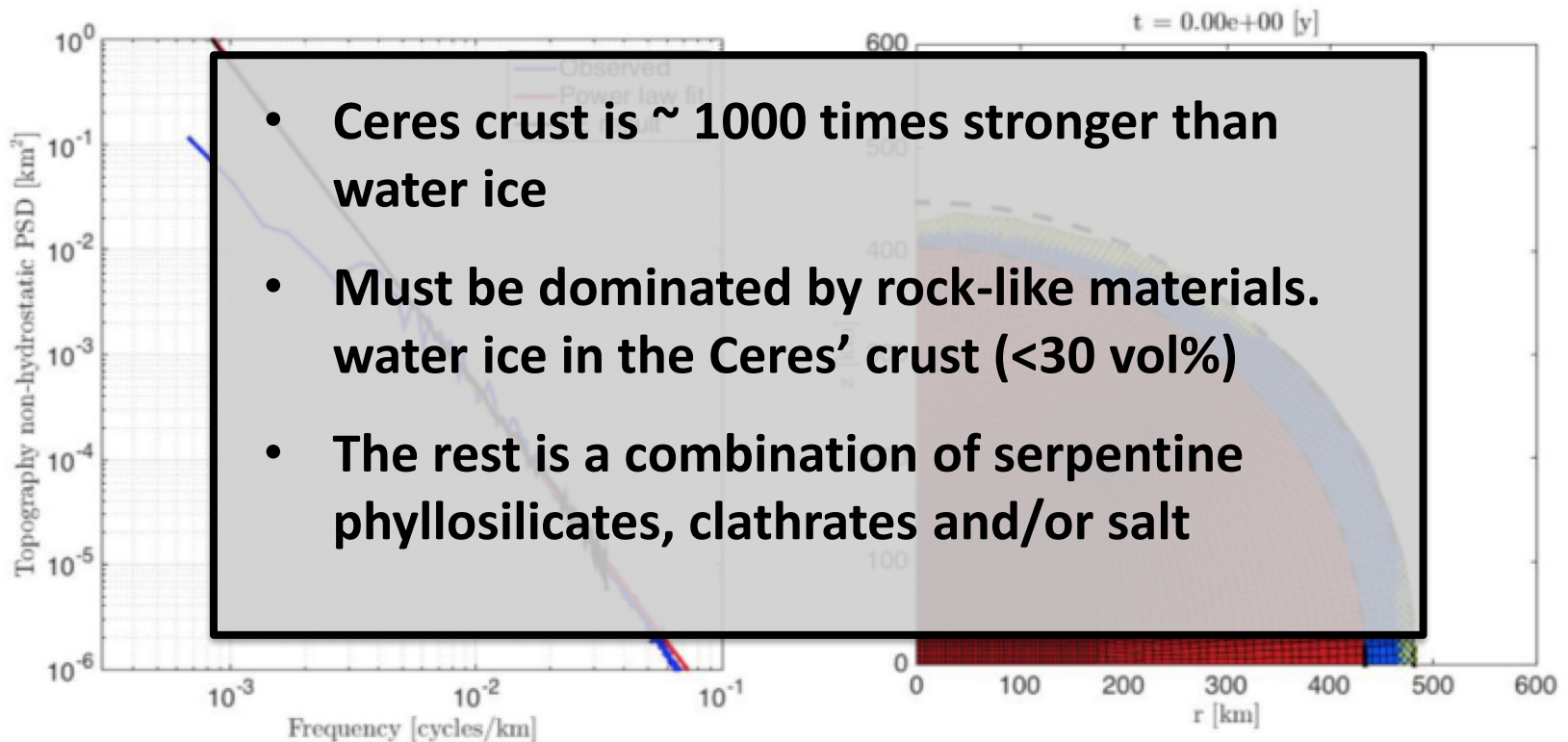
$$\nabla_i u_i = 0$$

- Compute the evolution of the outer surface power spectrum

Example of a FE modeling run



Finite element modeling results



Gravity and topography in spherical harmonics

- Shape radius vector

$$r(f, l) = R_0 \sum_{n=1}^{\infty} \sum_{m=0}^n (A_{nm} \cos(m l) + B_{nm} \sin(m l)) P_{nm}(\sin f)$$

- Gravitational potential

$$U(r, f, l) = \frac{GM}{R} + \sum_{n=2}^{\infty} \sum_{m=0}^n \frac{R_0^n}{r^{n+1}} (C_{nm} \cos(m l) + S_{nm} \sin(m l)) P_{nm}(\sin f)$$

- Power Spectral Density

$$S_n^{gg} = \sum_{m=0}^n \frac{C_{nm}^2 + S_{nm}^2}{2n+1}$$

gravity

$$S_n^{tt} = \sum_{m=0}^n \frac{A_{nm}^2 + B_{nm}^2}{2n+1}$$

topography

$$S_n^{gt} = \sum_{m=0}^n \frac{A_{nm} C_{nm} + B_{nm} S_{nm}}{2n+1}$$

gravity-topography
cross power

Isostatic model

Z_n - gravity-topography admittance

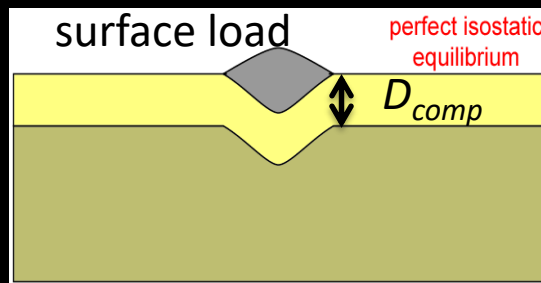
$$Z_n = \frac{S_{gt}}{S_{tt}}$$

➤ Linear two-layer hydrostatic model

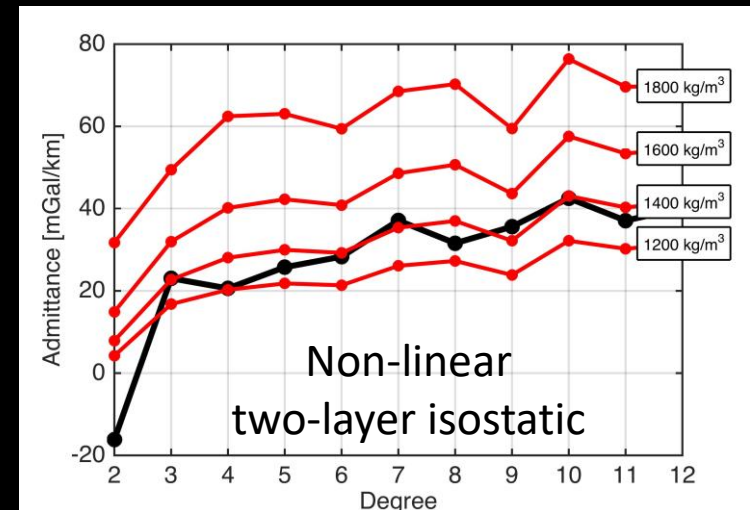
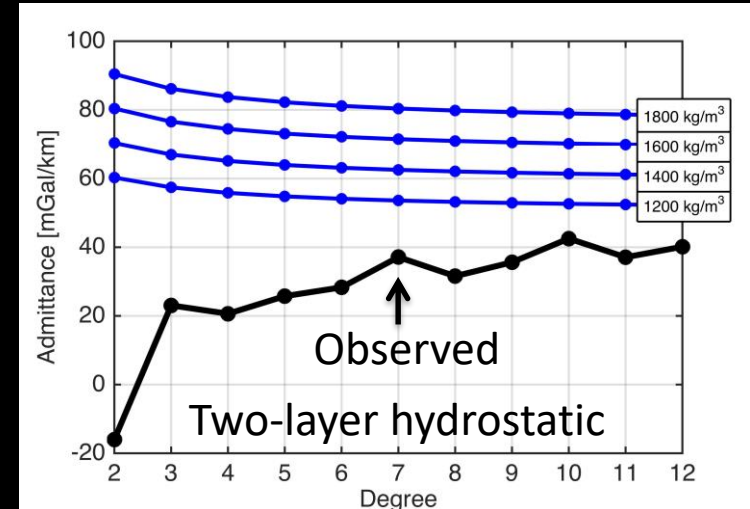
$$Z_n = \frac{GM}{R^3} \frac{3(n+1)}{2n+1} \frac{r_{crust}}{r_{mean}}$$

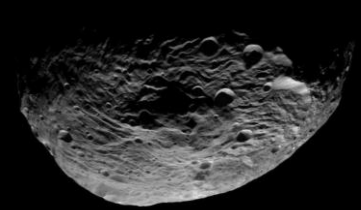
➤ Linear isostatic model

$$Z_n = \frac{GM}{R^3} \frac{3(n+1)}{2n+1} \frac{r_{crust}}{r_{mean}} \left(1 - \frac{D_{comp}}{R} \right)$$



D_{comp} - depth of compensation





Why Vesta?



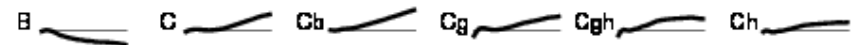
- Unique basaltic spectrum

Bus-DeMeo Taxonomy Key

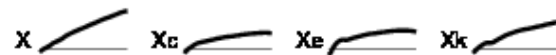
S-complex



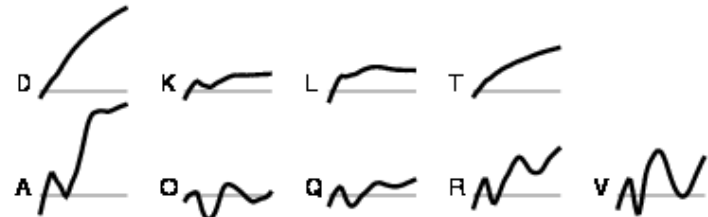
C-complex



X-complex



End Members



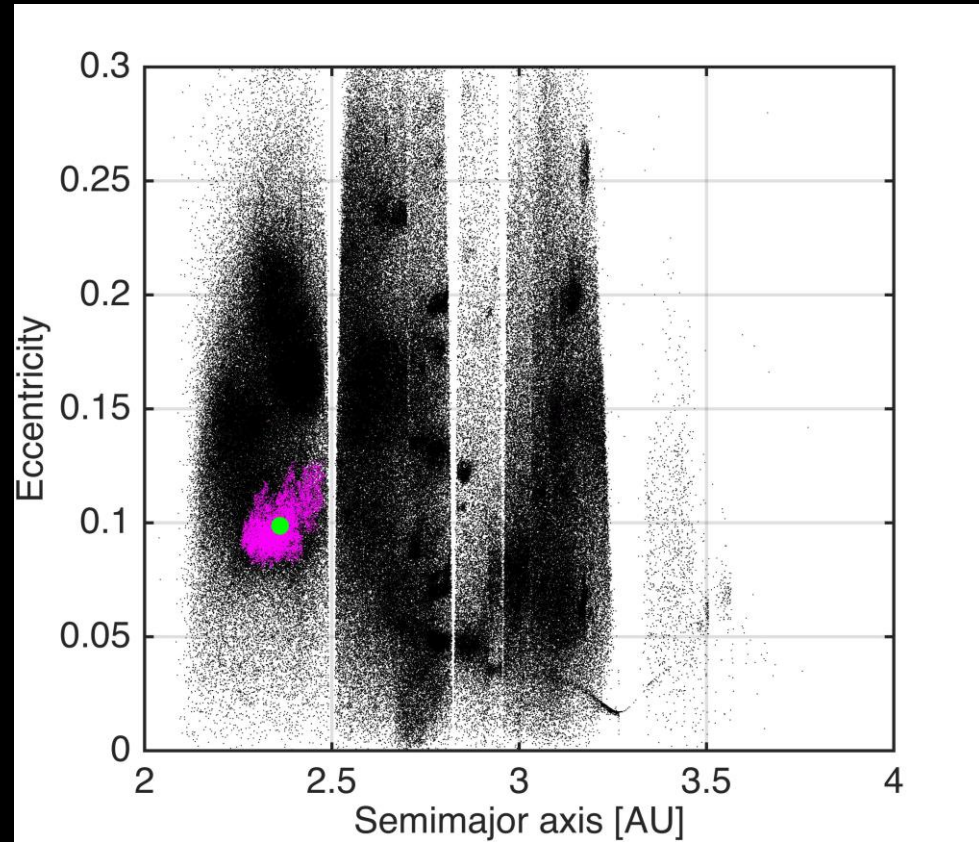
<http://smass.mit.edu/busdemeoclass.html>



Why Vesta?



- **Unique basaltic spectrum**
- **A group of asteroids in the dynamical vicinity of Vesta with similar spectra**



Why Vesta?

- Unique basaltic spectrum
- A group of asteroids in the dynamical vicinity of Vesta with similar spectra
- Large depression in the southern hemisphere of Vesta

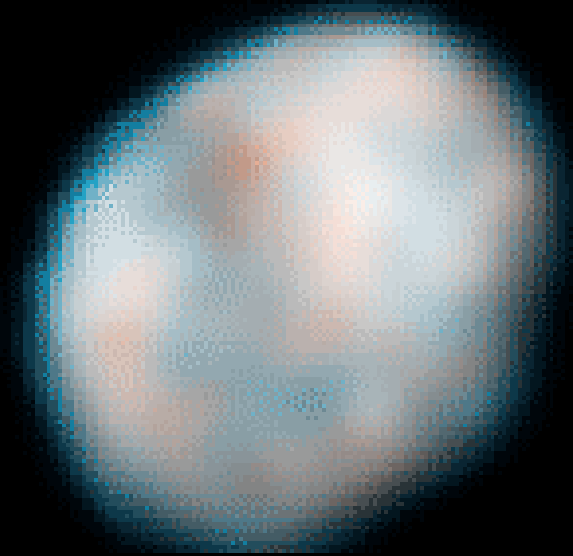
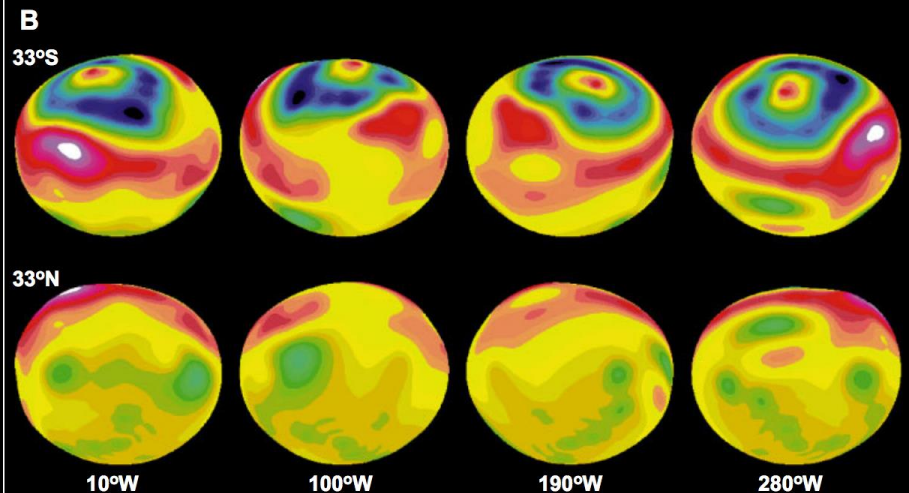


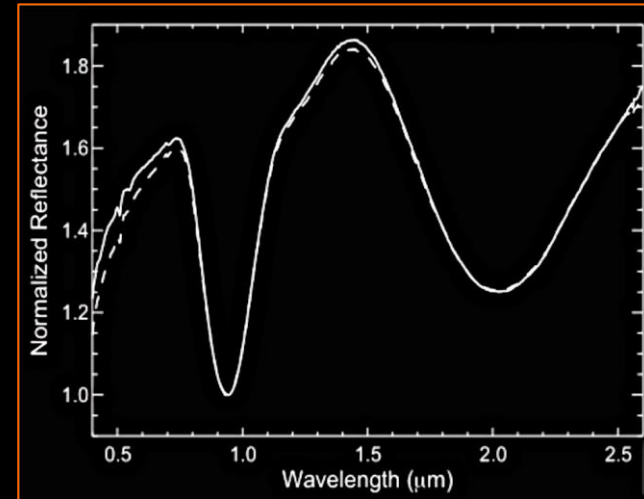
Image credit: NASA/HST



Thomas et al., 1997

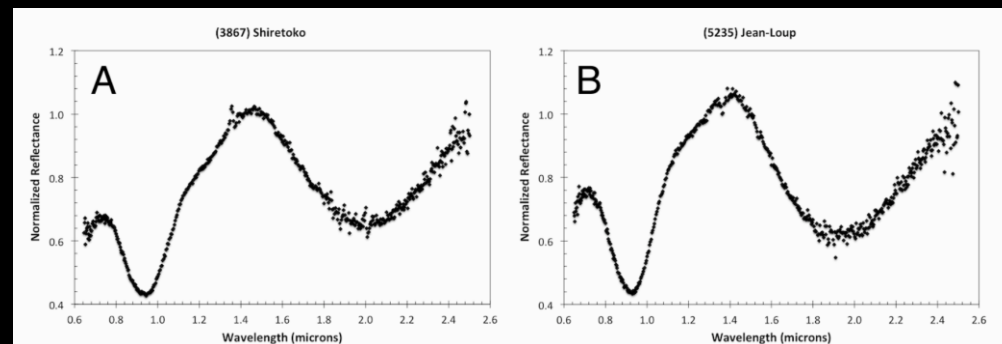
Why Vesta?

- Unique basaltic spectrum
- A group of asteroids in the dynamical vicinity of Vesta with similar spectra
- Large depression in the southern hemisphere of Vesta
- A group of Howardite-Eucrite-Diogenite (HED) meteorites, with similar reflectance spectra



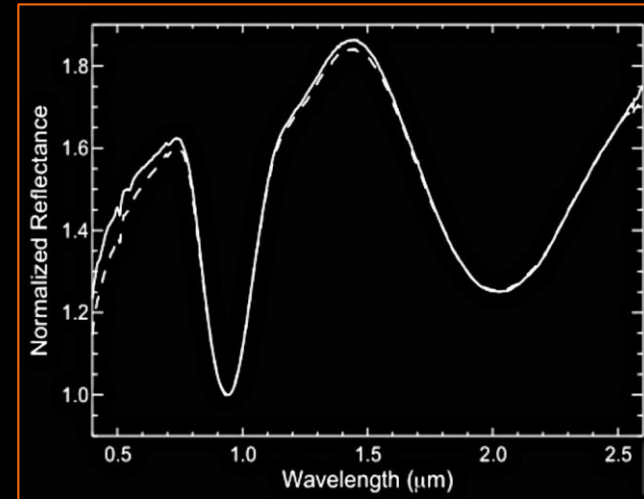
↑ Reflectance spectra of eucrite Millbillillie from Wasson et al. (1998)

↓ V-type asteroids spectra from Hardensen et al., (2014)



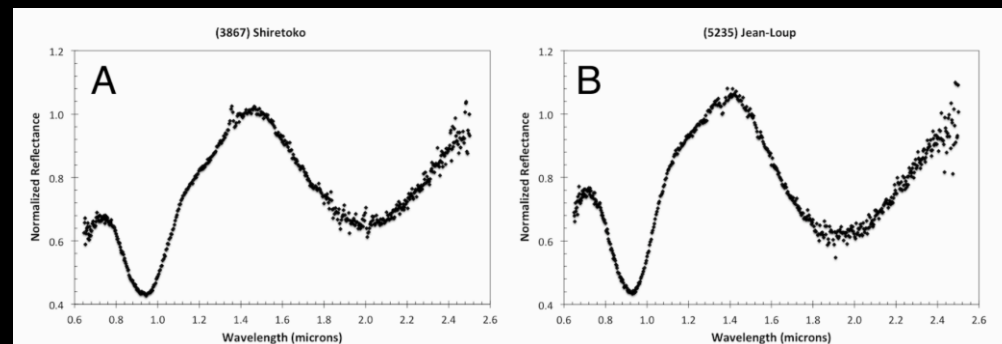
Why Vesta?

- Unique basaltic spectrum
- A group of asteroids in the dynamical vicinity of Vesta with similar spectra
- Large depression in the southern hemisphere of Vesta
- A group of Howardite-Eucrite-Diogenite (HED) meteorites, with similar reflectance spectra
- Strongest connection between a class of meteorites and an asteroidal family



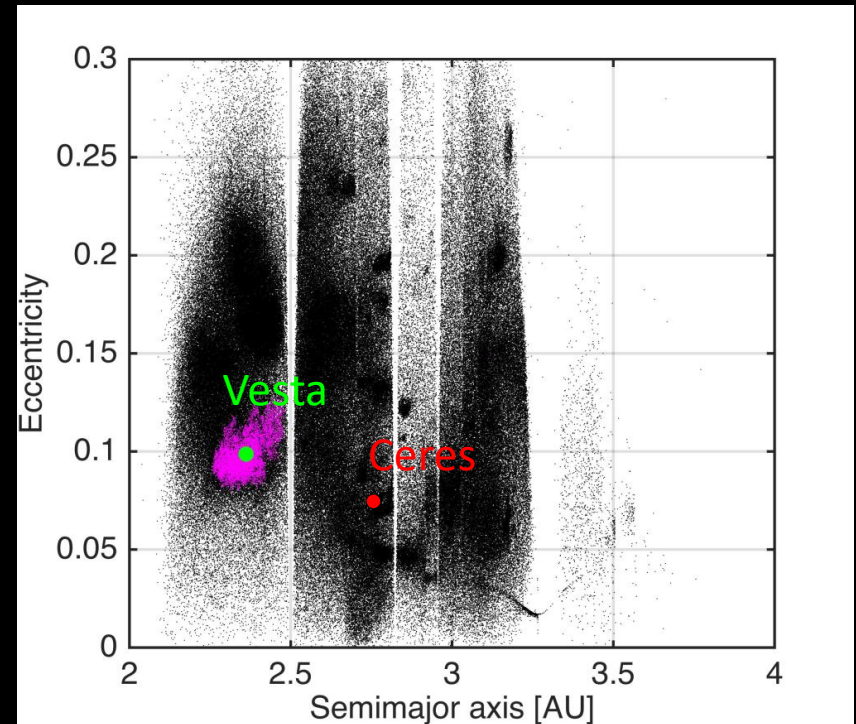
↑ Reflectance spectra of eucrite Millbillillie from Wasson et al. (1998)

↓ V-type asteroids spectra from Hardensen et al., (2014)



Why Ceres?

- Largest body in the asteroid belt
- Low density implies high volatile content
- Conditions for subsurface ocean
- Much easier to reach than other ocean worlds





What did we know before Dawn

- **Castillo-Rogez and McCord 2010**

Ceres accreted as a mixture of ice and rock just a few My after the condensation of Calcium Aluminum-rich Inclusions (CAIs), and later differentiated into a water mantle and a mostly anhydrous silicate core.

- **Zolotov 2009**

Ceres formed relatively late from planetesimals consisting of hydrated silicates.

- **Bland 2013**

If Ceres *does* contain a water ice layer, its warm diurnally-averaged surface temperature ensures extensive viscous relaxation of even small impact craters especially near equator

Note on Vening-Meinesz and Kaula rules

- Vening-Meinesz rule for variance of topography (Vening-Meinesz, 1951)

$$V_t \sim 1/n^2$$

- Kaula law for RMS of gravity (Kaula, 1963)

$$M_g \sim 1/n^2$$

- Are these two rules consistent assuming uncompensated topography?

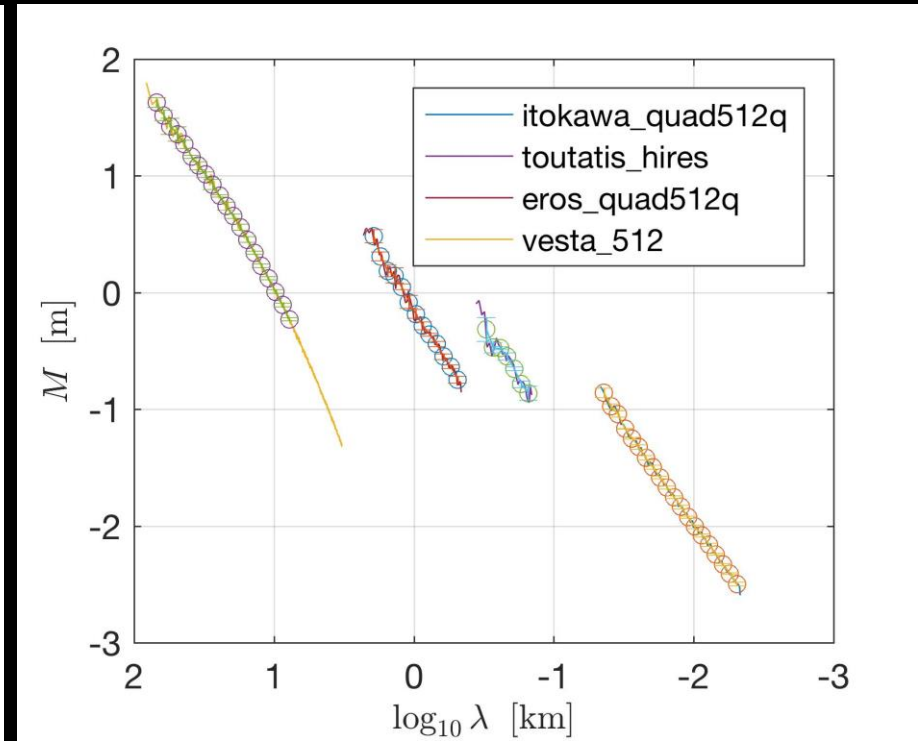
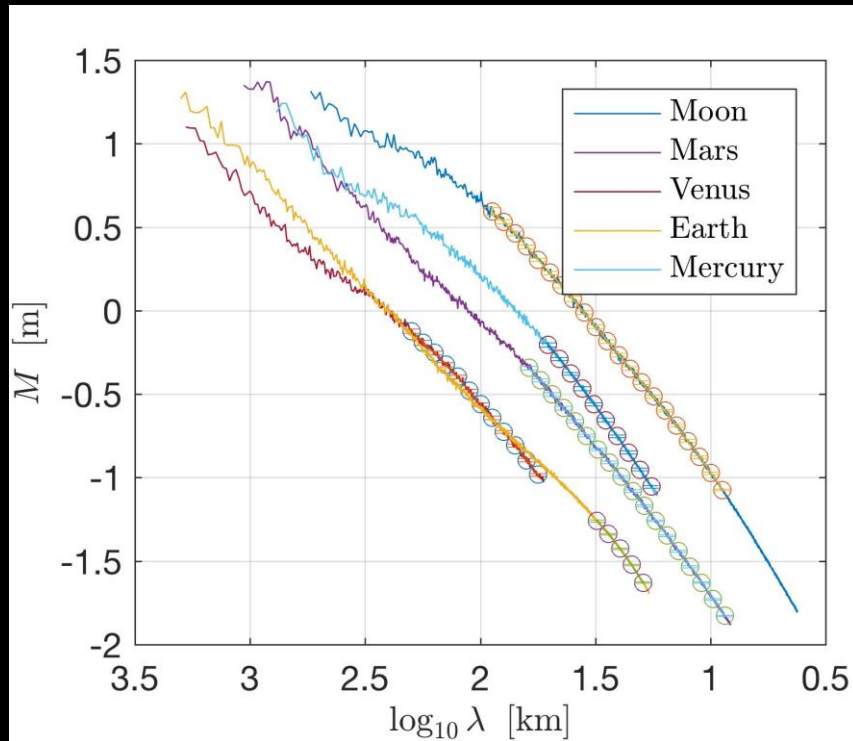
$$V_t \sim 1/n^2 \Rightarrow M_t \sim 1/n^{1.5} \Rightarrow M_g \sim 1/n^{2.5}$$

- But Kaula rule says $M_g \sim 1/n^2$ NOT $M_g \sim 1/n^{2.5}$



- Typically assumed in the literature Kaula and Vening-Meinesz rules are not mutually consistent assuming uncompensated topography

RMS spectra



Power laws

- General form of a power law

$$M = AR^{\alpha_1} \varrho^{\alpha_2} \lambda^{\alpha_3}$$

- Power law assuming (inverse) surface gravity scaling ($g \sim R^* \rho$)

$$M = AR^{-1} \varrho^{-1} \lambda^{\alpha_3}$$

- If we take a \log_{10} of M , we get an equation of a hyperplane

$$\log_{10} M = \log_{10} A + \alpha_1 \log_{10} R + \alpha_2 \log_{10} \varrho + \alpha_3 \log_{10} \lambda$$

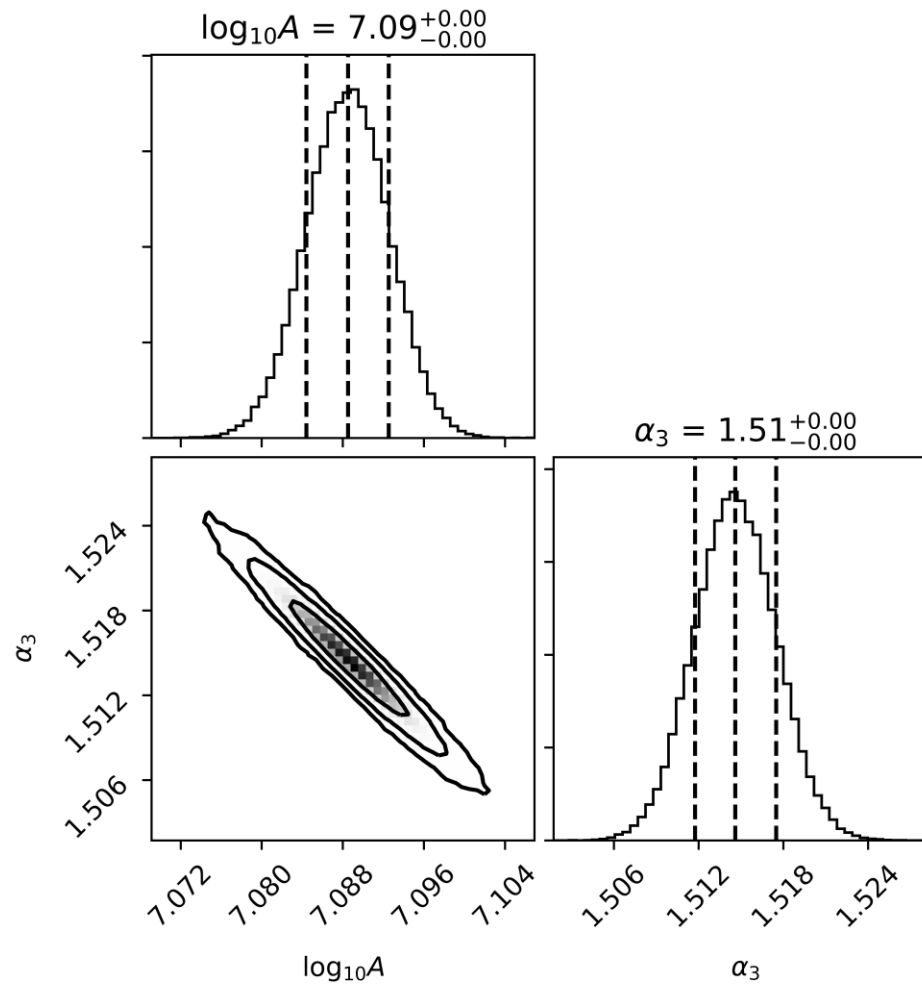
- In our data set, we have a lot of points along the λ direction and not as many points on the other two (R and ρ) directions.
 - In the R and ρ directions, we have as many data points as we have bodies
 - In the λ direction, we have as many data points as many we have λ bins.

Markov-chain Monte-Carlo (MCMC)

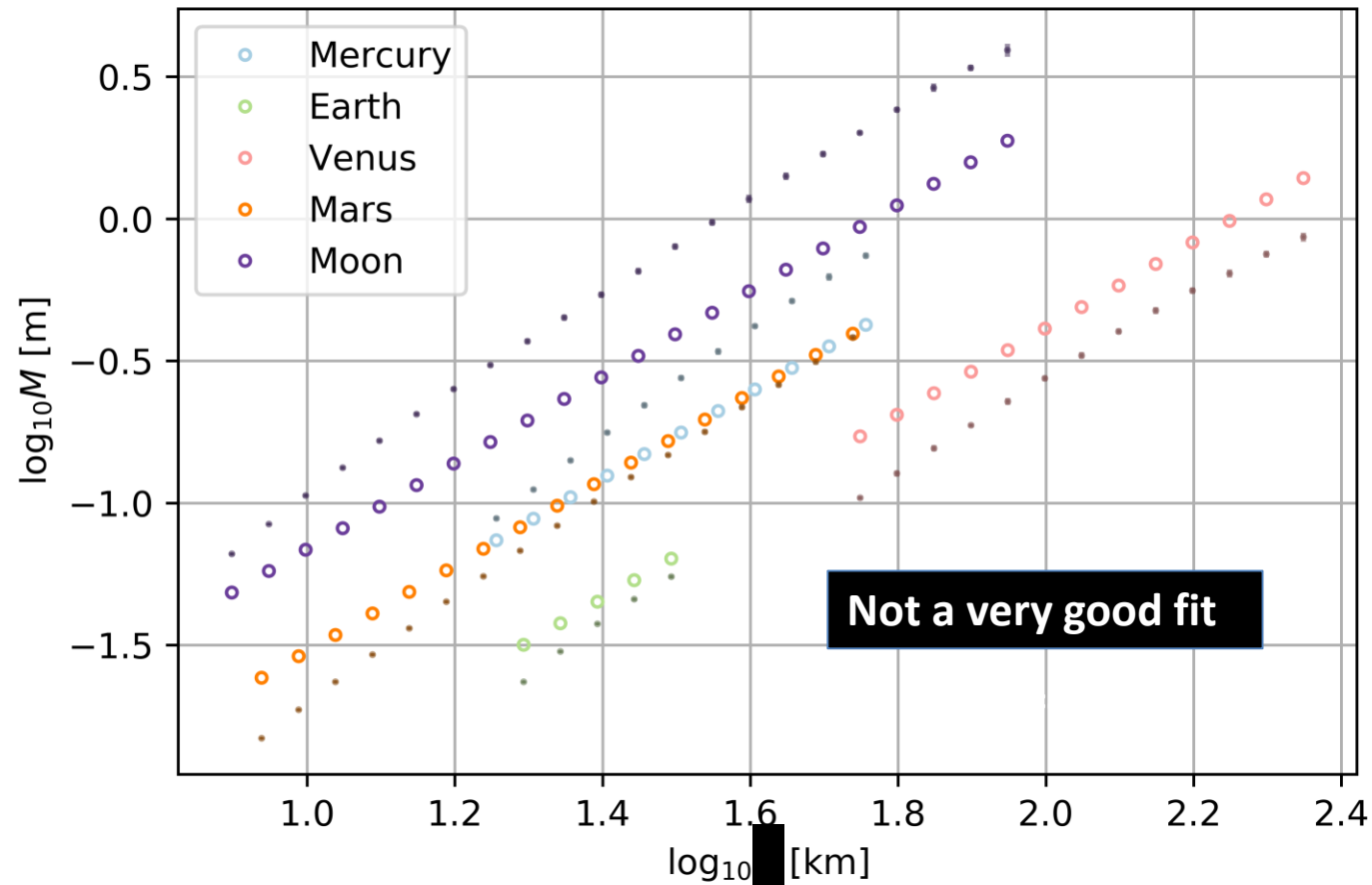
- We use a free Python library `emcee` (Foreman-Mackey et al., 2013) to find the best-fit parameters of a power law.
- `emcee` library is based on *Affine Invariant Markov chain Monte Carlo sampler* (Goodman and Weare, 2010)
- We fit a power law model with:
 - two parameters: A , α_3 -- assuming surface gravity scaling ($\alpha_1=-1$, $\alpha_2=-1$)
 - four parameters: A , α_1 , α_2 , α_3 -- general scaling.
- For each MCMC run, we will show:
 - A triangle plot of the posterior distribution of the model parameters. This allows seeing the covariances between the parameters.
 - A plot of best-fit model versus the observations. We also show a reduced chi squared value to judge about the quality of the best-fit.
- `emcee` is an extensible, pure-Python implementation of

Results of the MCMC runs

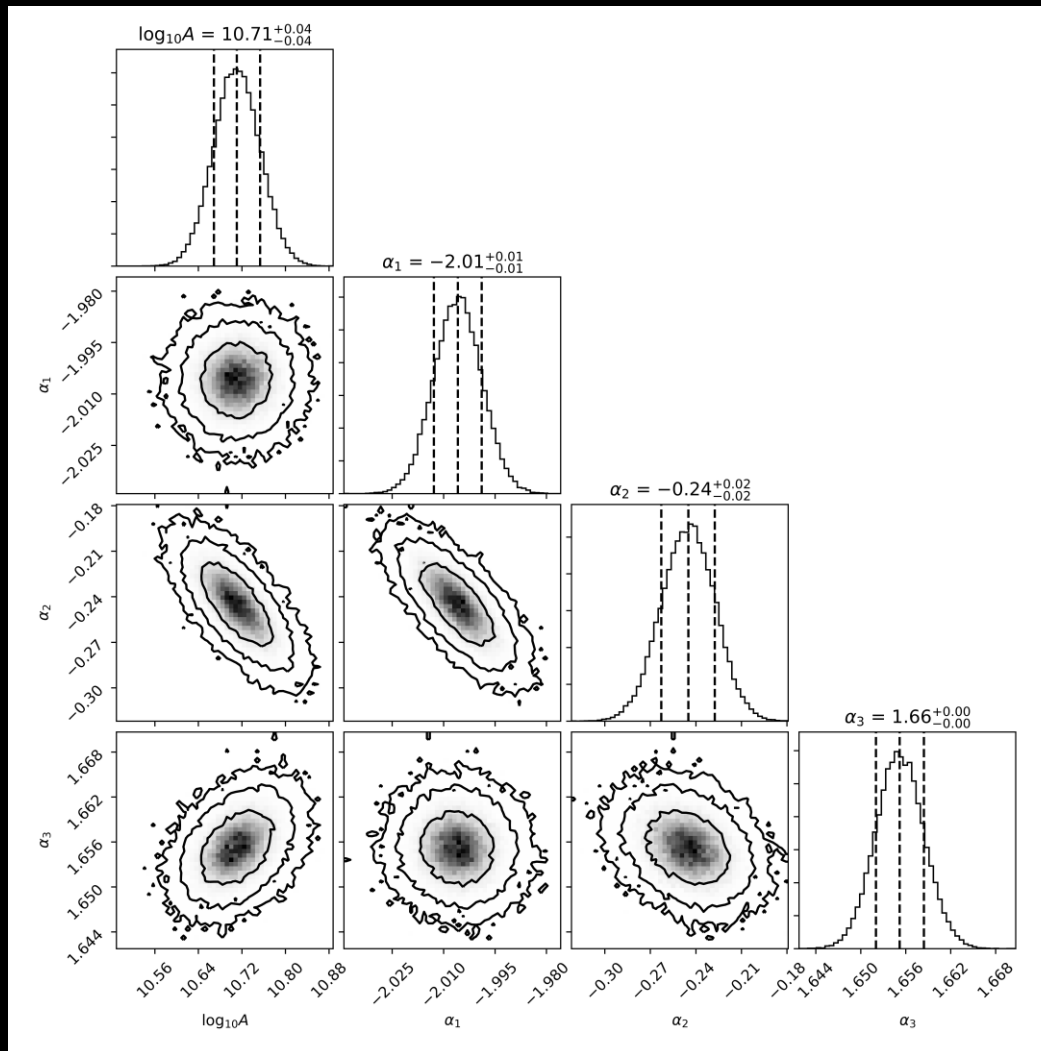
Planets, gravity scaling



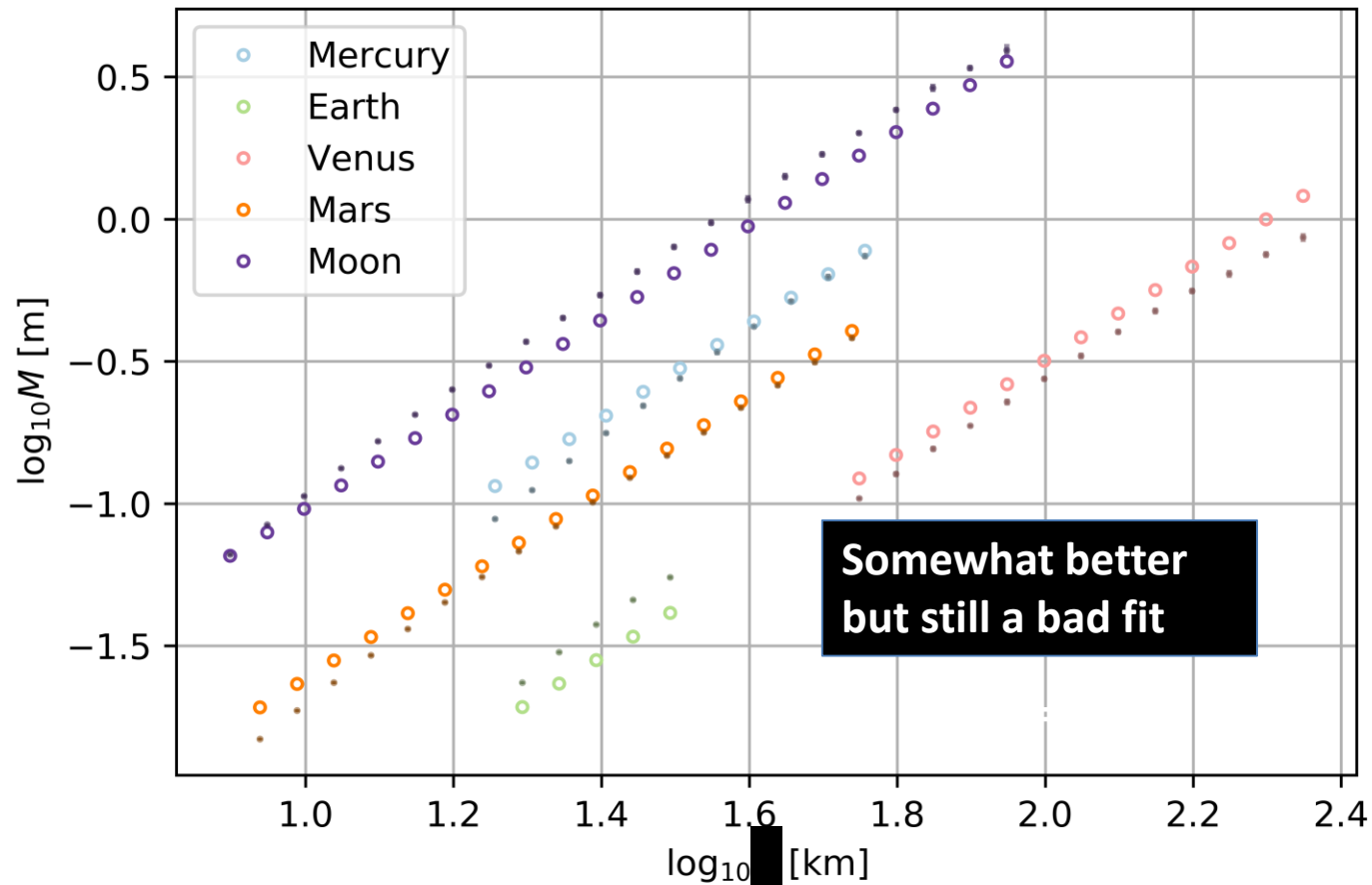
Planets, gravity scaling



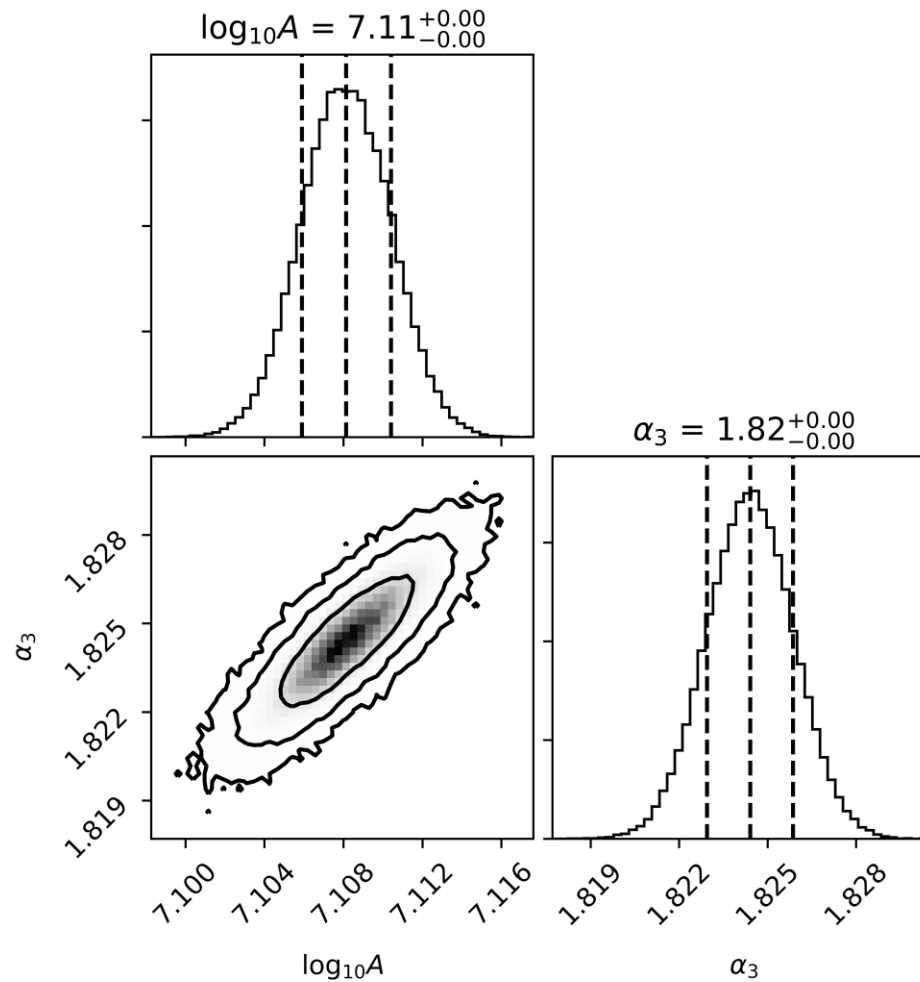
Planets, general scaling



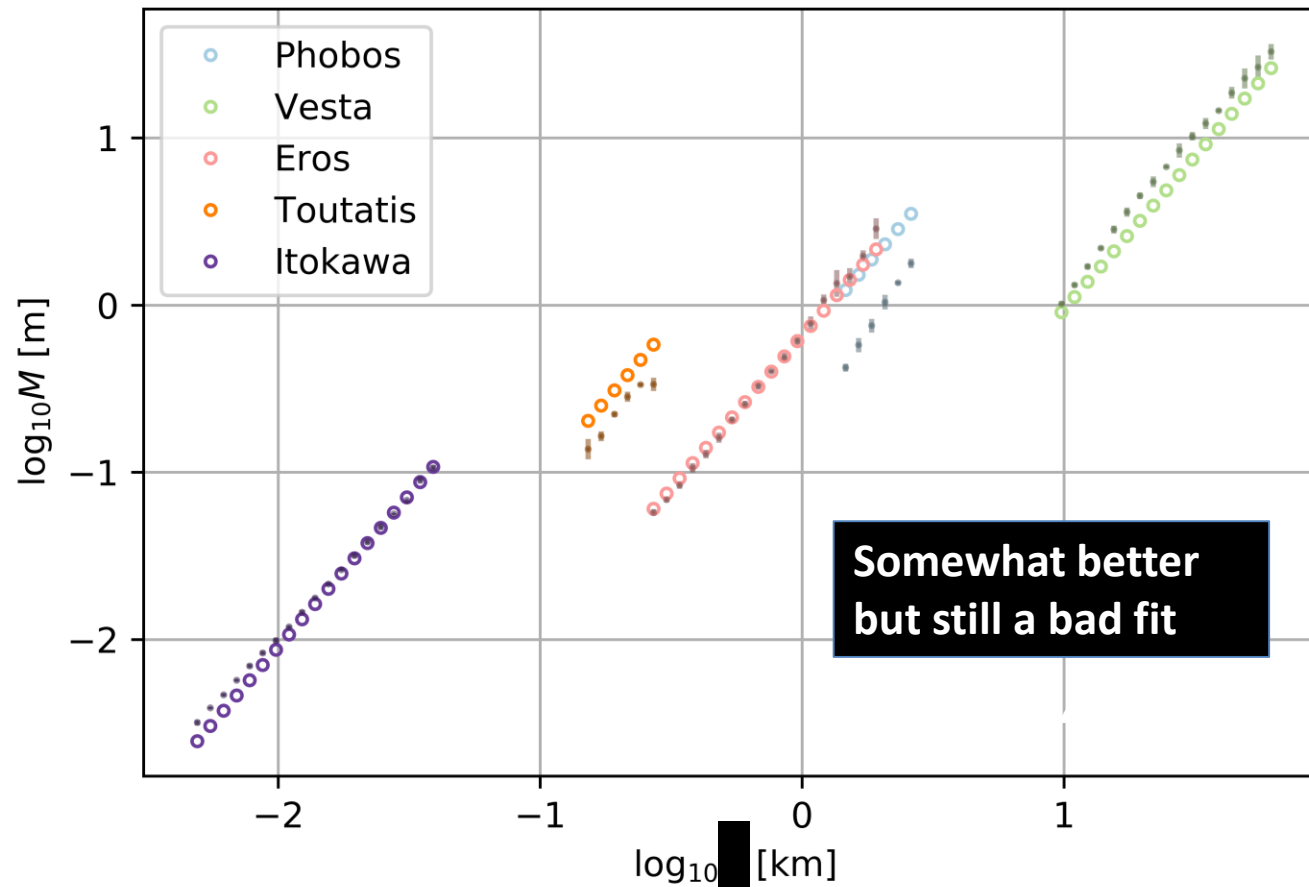
Planets, general scaling



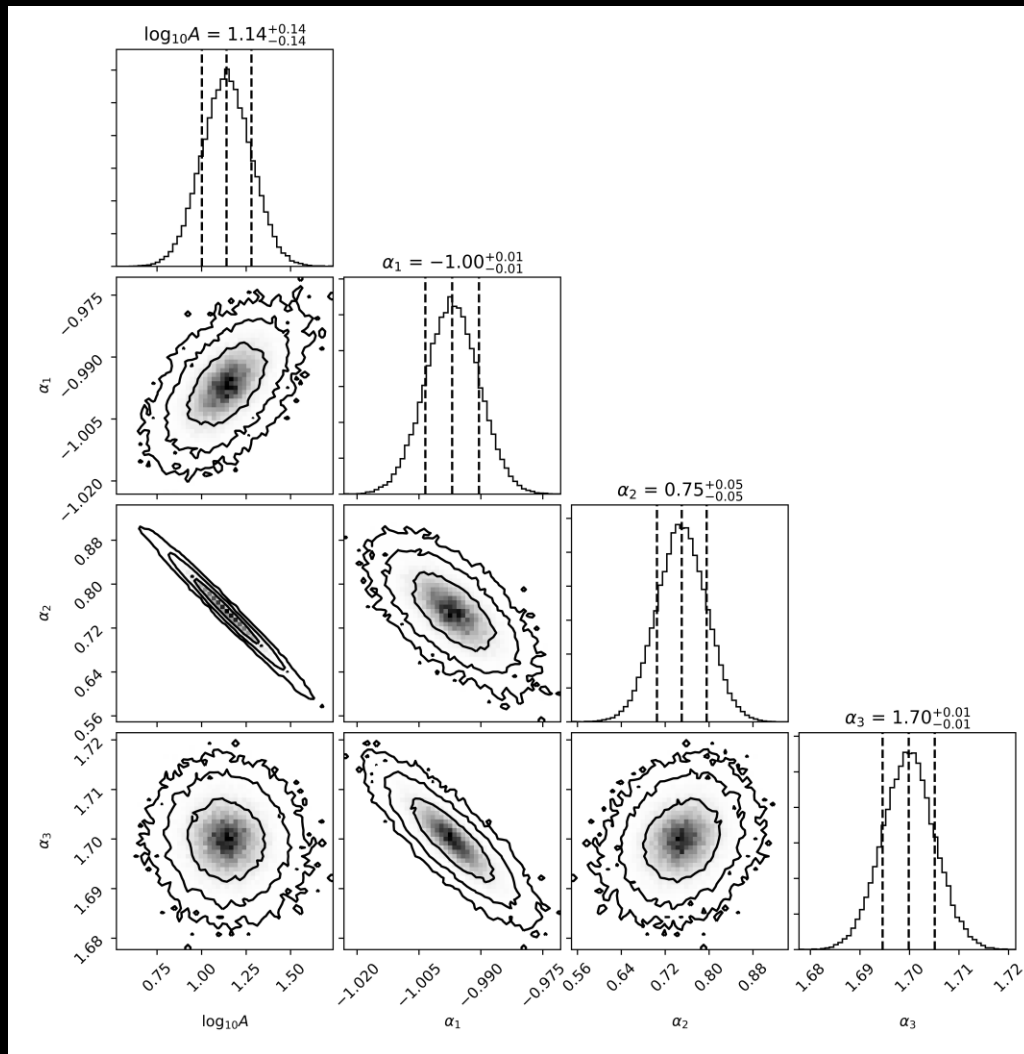
Asteroids, gravity scaling



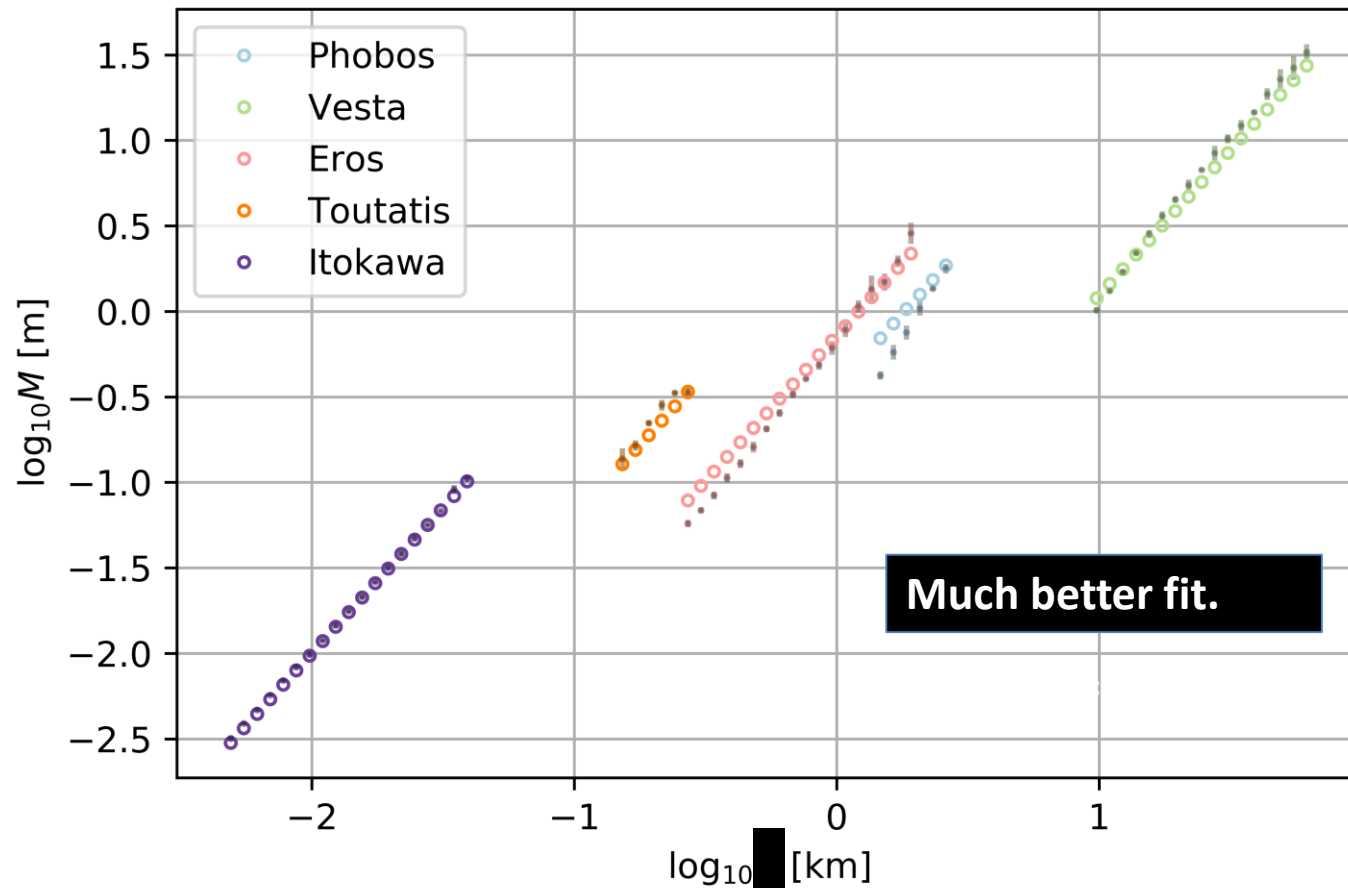
Asteroids, gravity scaling



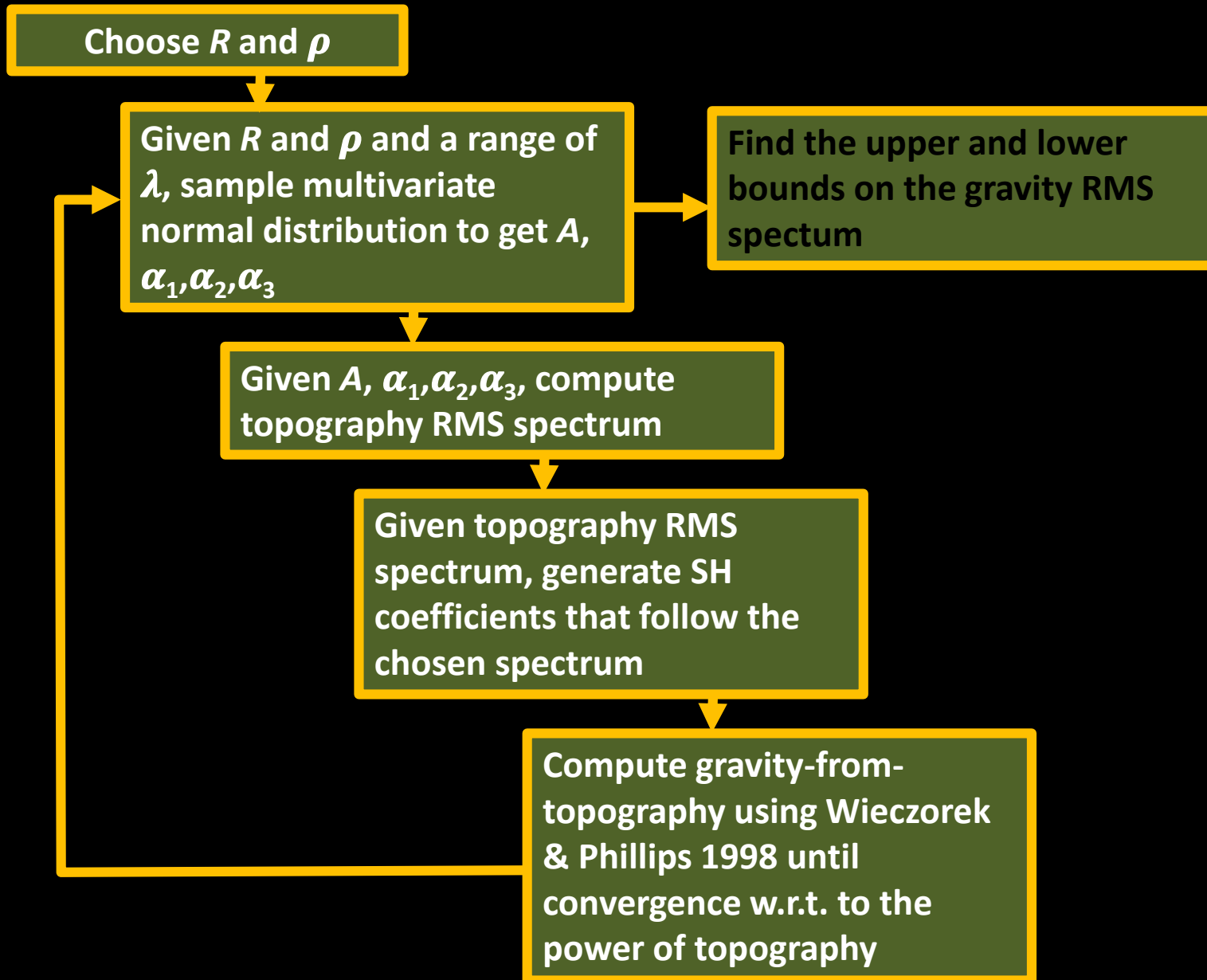
Asteroids, general scaling



Asteroids, general scaling



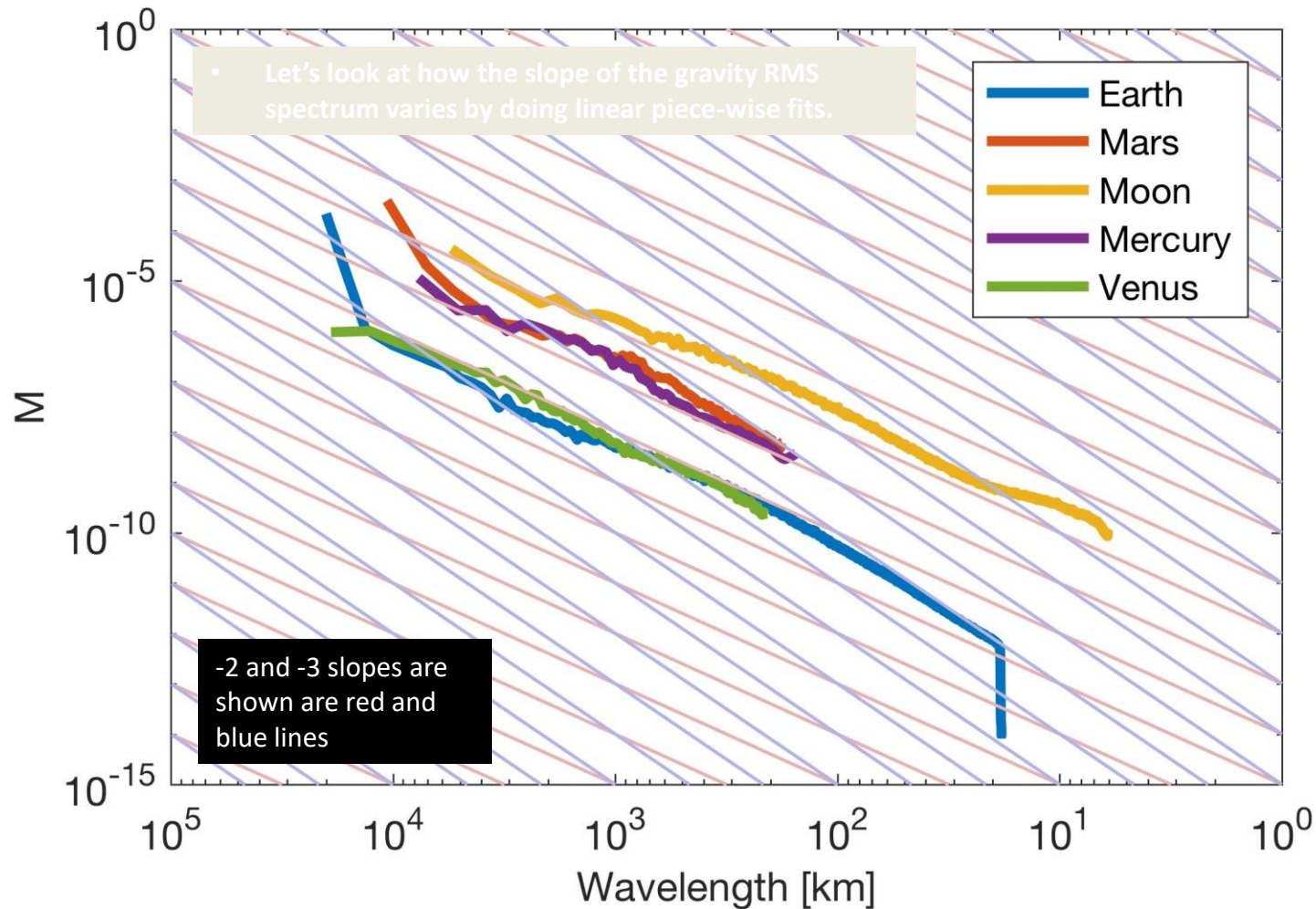
A priori constraint on gravity RMS



Summary

- Topography RMS spectra of 4 terrestrial planets and the Moon cannot be simultaneously fit with a single power law of the gravity-scaling or general form.
- Topography RMS spectra of asteroids **CANNOT** be *satisfactorily* fit with a power law the gravity-scaling form.
- Topography RMS spectra of asteroids **CAN** be *satisfactorily* fit with a power law of the general form.
- Despite having different internal structure, composition and mechanical properties of the surface layer, the asteroid topography spectra can be effectively modeled as a general power law

Gravity RMS spectra



Slopes of piecewise fitted gravity RMS spectra

